Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

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Abbreviations of units follow the standard of Abbreviations for Scientific and Engineering Terms of the Am. Standards Assoc. Examples: psi (pounds per square inch); cps (cycles per second); mph (miles per hour).

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Applied Mechanics Reviews

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August 1948 Vol. 1, No. 8

General Theoretical and Experimental Methods

(See also Revs. 1174, 1219)

1169. W. Klafke, "The tensor scale (Die Tensorwaage)," *Ingen.-Arch.*, 1947, vol. 16, no. 1, pp. 5–13.

The author derives the formulas that express the direction and magnitude of the principal values of a plane tensor in terms of three components in directions 120 deg apart. Special attention is paid to the case of the plane strain tensor. The components are then strains measured in equally spaced directions, and to be computed are the direction and magnitude of the principal stresses. Often more than three equiangular strains are measured and the computation must then be performed by least squares.

A mechanical device, the "tensor scale," is described that solves this last, rather involved problem. The beam of the scale carries on one side of the knife-edge a revolving circular disk, and at the other side two sliding weights. On the disk there are attached at the corner-points of a regular polygon horizontal pins on which little weights proportional to the measured strains can be fixed. The positions of equilibrium of the disk show the principal directions, whereas the principal values can be found by balancing the scale at two positions of the disk by sliding the weights. An additional device permits also the introduction of negative strains. The desirable dimensions of the instrument are discussed.

A. van Wijngaarden, Holland

1170. W. J. Duncan, "Assessment of errors in approximate solutions of differential equations," *Coll. Aero. Cranfield Rep.*, no. 13, Dec. 1947, pp. 1–9.

This report discusses the errors involved in certain numerical processes used in obtaining approximate solutions of differential equations. It is shown that whenever the Green's function of a linear problem is one-signed (for instance, Poisson's equation), or whenever the Green's function itself is approximately known, or whenever a step-by-step process possessing an index (for instance, Runge-Kutta) is used, the magnitude of the error in the approximate solution obtained can be assessed.

Stephen H. Crandall, USA

1171. Louis Brand, "Vector and tensor analysis," John Wiley and Sons, Inc., New York, N. Y., 1948. Cloth, 8.5 × 5.5 in., 439 pp., 187 figs., price not shown.

This text amply satisfies the needs of engineering and physics students, and also contains some purely mathematical material, such as theories of dyadics, motors, and quaternions (the physicist at large refuses to recognize their usefulness and favors tensor calculus which taxes his memory less). There is also more differential geometry in the text than the physicist would insist upon.

Apart from the quaternions and motors, the choice of subject matter is quite conservative. As usual, no attempt is made to take notice of the modern (abstract algebraic) definition of a vector, so useful to the engineer. The antique "directed magnitude" definition is retained (note that it makes a light beam a vector, but does not cover visual colors whose vector treatment is well established). Neither is an attempt made to define covariant vectors (not merely covariant *components*), even if the phrase "contravariant tensor" appears on p. 370.

The formal apparatus of the calculi discussed is presented clearly and neatly. Many examples and carefully organized exercises reward the reader with valuable information for the pains taken in solving them.

A. W. Wundheiler, USA

1172. E. H. Neville, "Ill-conditioned sets of linear equations," *Phil. Mag.*, Jan. 1948, vol. 39, pp. 35–48.

The author demonstrates by a numerical example that iterative processes used intelligently in the case of an ill-conditioned set of linear equations are superior to the escalator processes suggested by J. Morries [*Phil. Mag.*, 1946, vol. 37, p. 106].

Wei-Zang Chien, China

General Dynamics, Kinematics, Friction

(See also Revs. 1180, 1265)

1173. O. Günther, "The performance of railroad vehicles in curves and its influence on their design, especially on that of steam locomotives (Der Bogenlauf von Eisenbahnfahrzeugen und sein Einfluss auf ihre Gestaltung, insbesondere auf die von Dampflokomotiven mit Schubachsen)," *Glasers Ann.*, Sept. 1947, vol. 71, pp. 167–178.

The Heumann method of determining forces on locomotive wheels is applied to several types of locomotives going around curves, with emphasis on designs having side-play in one or more axles. The Heumann diagrams are shown and numerical results are tabulated.

The locomotives investigated are (1) 0-8-0 with rigidly mounted axles, (2) 0-8-0 with ± 1-in. side-play of the second and fourth axles, with and without return springs, (3) 2-8-2 with Helmholtz-type trucks and side-play in the first and last drive wheel axles, (4) same as (3) but with truck design improvements, (5) 2-8-2 with all wheels spring-mounted. Wheel and frame forces for various radii of the curve and for several forward and reverse speeds are computed at entrance into, passage through, and exit from a curve. The calculated wear and safety indexes confirm reported operating experience and demonstrate the advantages of side-play and elastic designs over rigid construction with respect to wheel wear, shaking forces, and safety.

Readers should be familiar with the Heumann method of minima [see papers by Heumann and by Nöthen, *Lokomotive*, 1942, pp. 1 and 129] to be able to follow the semigraphical force analyses of this paper.

G. A. Nothmann, USA

1174. L. Statham, "A pickup for the measurement of quasi-static angular acceleration," *Rev. sci. Instrum.*, June 1948, vol. 19, pp. 381–384.

This paper describes an angular accelerometer in which the stiffness is supplied by four independent filaments of resistance wire. The resistance wires are connected electrically in the form of a bridge circuit. Change in position of the accelerometer rotor results in electrical unbalance of the bridge circuit thereby giving a proportional indication of the angular acceleration.

C. E. Warren, USA

1175. Henry Favre, "A course in mechanics, vol. II: dynamics of rigid bodies (Cours de mécanique. II. Dynamique des corps solides rigides)," Dunod, Paris, France, 1947. Paper, 6.2 × 9 in., 434 pp., 164 figs., price not shown.

This volume contains the kinematics and dynamics of a particle and of a rigid body. The selection of topics is conventional, their treatment is in the well-established French tradition. Vector symbolics are used with moderation. There are exercises at the end of each chapter (without answers). The presentation is clear and neat as is usual in this category of French texts.

A. W. Wundheiler, USA

Vibrations, Balancing

(See also Revs. 1213, 1219, 1234)

1176. G. Horvay, "Chordwise and beamwise bending frequencies of hinged rotor blades," *J. aero. Sci.*, Aug. 1948, vol. 15, pp. 497–502.

The author gives a formula expressing an arbitrary natural bending vibration frequency of a stationary blade as a constant times the first natural frequency. For the rotating blade he expresses an arbitrary natural frequency as a simple function of the corresponding frequency for the nonrotating blade plus a constant times the angular velocity of rotation. He then determines the constants involved for several natural frequencies of a particular blade, and states that these constants are rather insensitive to variations of shape and mass of the blade. If this is true, the higher frequencies of a rotating blade may, in general, be obtained to a fair degree of accuracy from the fundamental frequency of the nonrotating blade alone. N. O. Myklestad, USA

1177. I. N. Vekua, "On a method for boundary-value problems on sinusoidal vibrations of an elastic cylinder" (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, May 11, 1948, vol. 60, pp. 779–782.

The author considers a class of boundary-value problems associated with the transverse vibrations of an infinitely long, elastic cylinder. The displacement field is assumed to be of the form

$$u = U(x, y)\begin{Bmatrix}\cos \nu t\\ \sin \nu t\end{Bmatrix}, \quad v = V(x, y)\begin{Bmatrix}\cos \nu t\\ \sin \nu t\end{Bmatrix}, \quad w = 0$$

where the x,y-plane is taken perpendicular to the axis of the cylinder. The values of U, V are prescribed on the boundary of the simply connected cross section.

The approach used is based on an extension of Muskhelishvili's method for the plane problem in elasto-statics, given by the author in an earlier paper [*C. R. Acad. Sci. URSS (Doklady Akad. Nauk SSSR)*, 1937, vol. 16, no 3]. The problem is reduced to the solution of two singular integral equations which, in turn, can be reduced to a system of equations of the Fredholm type.

E. Sternberg, USA

1178. R. H. Scanlan, "A note on transverse bending of beams having both translating and rotating mass elements," *J. aero. Sci.*, July 1948, vol. 15, pp. 425–426, 434.

The usual equations of transverse vibration of beams, taking into consideration only translation of the distributed mass without rotation, take the form

$$\omega^{-2}[y] = [\delta_F]\ [m]\ [y]$$

where $[\delta_f]$ is the matrix of deflection influence coefficients for transverse forces. The author extends this matrix equation to include the effect of rotatory inertia perpendicular to the axis of the beam by introducing the deflection influence coefficient δ_M due to moment, and the slope influence coefficients σ_F and σ_M due to force and moment respectively. The matrix equation then takes the form

$$\omega^{-2}\begin{bmatrix} y \\ y' \end{bmatrix} = \begin{bmatrix} \delta_F, & \delta_M \\ \sigma_F, & \sigma_M \end{bmatrix}\begin{bmatrix} m \\ I \end{bmatrix}\begin{bmatrix} y \\ y' \end{bmatrix}$$

which is interpreted by the equations

$$\omega^{-2}y_i = \sum_{j=1}^{n} m_j y_j \delta_F(i,j) + \sum_{j=1}^{n} I_j y_j' \delta_M(i,j)$$

$$\omega^{-2}y_i' = \sum_{j=1}^{n} m_j y_j \sigma_F(i,j) + \sum_{j=1}^{n} I_j y_j' \sigma_M(i,j)$$

where y_i' is the slope of the beam at i.

The elastic matrix for this refinement has a maximum of four times the number of elements necessary for the pure translational case. However, the rotatory inertia of masses on the axis of the beam is generally negligible compared to the rotatory inertia of masses displaced from the axis, and the computational labor is seldom increased fourfold.

The method is illustrated by an example of a uniform cantilever beam carrying four unequal masses, the one at the tip being displaced from the axis of the beam to introduce a large rotatory inertia. For this example the natural frequency is lowered by 3 and 6 per cent for the two positions of the end mass.

W. T. Thomson, USA

Wave Motion, Impact, Seismology

(See also Revs. 1182, 1185, 1248, 1286, 1291)

1179. Z. Sekera, "Helmholtz waves in a linear temperature field with vertical wind shear," *J. Met.*, June 1948, vol. 5, pp. 93–102.

The theory of short gravity waves in the atmosphere is extended to the case where, in addition to a temperature gradient, there is a gradual transition of the (horizontal) wind velocity. Temperature and wind velocity are functions only of the height. The fundamental equations, derived by the perturbation method, lead to an ordinary second-order differential equation. The case where the temperature and velocity gradients are both constant is discussed, qualitatively, in some detail; quantitative results are promised.

It is shown that the conditions for the existence of waves imply that wave speed and wind speed must be equal at some level within the transition layer, and that, in the neighborhood of this level, the lapse rate must be adiabatic or superadiabatic. The conclusions are in agreement with observations on "billow clouds." W. G. Bickley, England

1180. **B. Milwitzky, "A generalized theoretical investigation of the hydrodynamic pitching moments experienced by V-bottom seaplanes during step-landing impacts and comparisons with experiment,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1630, June 1948, pp. 1–62.

A theoretical method of analysis of the hydrodynamic pitching moments experienced by V-bottom seaplanes during step-landing impacts is presented. It is shown that the pitching moment and the center of pressure may be represented by dimensionless variables which account for such factors as gross weight, angle of dead rise, trim angle, and initial velocity. These variables are governed by the magnitude of the approach parameter which depends only on the trim and the initial flight-path angle.

The results of the study show that the maximum hydrodynamic pitching moment is attained slightly after maximum acceleration is reached, and prior to the point of maximum draft. It is also determined that the moment about the step at any given stage of impact is independent of the angle of dead rise. During immersion the center of pressure is located at a distance only slightly greater than one third the wetted length forward of the step, converging to a limiting value of exactly one third as the planing condition is approached.

Equations are presented which analyze the entire immersion process from the instant of initial contact until the seaplane rebounds from the water, through a range of flight paths from planing to steep impacts where the resultant velocity is normal to the keel. Ernest G. Stout, USA

1181. **V. V. Sokolovski, "Propagation of cylindrical shear waves in an elastic-viscous-plastic medium"** (in Russian), *Notes Acad. Sci. USSR* (*Doklady Akad. Nauk. SSSR*), June 11, 1948, vol. 60, pp. 1325–1328.

Problems of one-dimensional wave propagation in inviscid elastic-plastic materials have been studied extensively during recent years. The present paper is concerned with the influence of the viscosity of the material on phenomena of this kind. To keep the mathematical work as simple as possible, the author investigates a problem with axial symmetry using cylindrical co-ordinates. The radial and axial velocity components are assumed to vanish, and the circumferential velocity component is assumed to depend only on the radius r and the time t.

The shearing stress τ and the shear strain γ are supposed to be related to each other by $\mu d\gamma/dt = d\tau/dt$ if $|\tau| \leq \tau_o$ (μ = coefficient of viscosity, τ_o = yield stress in shear) and by $\mu d\gamma/dt = d\tau/dt + k(|\tau| - \tau_o)$ sign τ if $|\tau| \geqslant \tau_o$. The general equations for the propagation of cylindrical shear waves are established. It is shown how these equations can be integrated by means of the method of characteristics. The following example is treated explicitly: The surface of a cylindrical hole in an infinite body is suddenly exposed to a shearing stress $\tau_{r\theta}$ which then decreases linearly with time; cylindrical shear waves caused by this surface stress are determined. W. Prager, USA

Acoustics

1182. **Isadore Rudnick, "The propagation of an acoustic wave along a boundary,"** *J. acoust. Soc. Amer.*, Mar. 1947, vol. 19, pp. 348–356.

This paper is concerned with the sound field of an acoustic source in the neighborhood of a plane boundary between two semi-infinite media. Obviously, the usual plane wave theory can no longer be applied to such a limiting case. Following the methods of Sommerfeld, Van Der Pol, and Norton for treating the electromagnetic field of a dipole similarly situated, the author obtains a rigorous solution in terms of the infinite integral of the Bessel function, which is typical for cylindrical waves. As pointed out by Van Der Pol, this solution may be considered as that due to a point source and a diffuse image through reflection. This procedure seems very useful in attacking many boundary-value problems.

Owing to the difficulty of expressing this integral in a closed form, some approximate analytic solutions have been obtained for two cases, when the second medium is: (a) highly sound-absorbing (porous) with low frequency, and (b) non-sound-absorbing (nonporous). The calculation for the source point right at the boundary of the Quietone, with air above, shows a few interesting results: (1) There is a minimum of sound pressure at some distance above the boundary. (2) Above this minimum, the pressure amplitude decreases rapidly with distance from the boundary. (3) The phase lag of the reflected wave behind the free field wave decreases with distance above the boundary. (4) If the source is far away from the boundary, the solution confirms that of the plane wave theory.

Chieh-Chien Chang, USA

1183. **W. Tak, "Measuring reverberation time by the method of exponentially increasing amplification"** (in English), *Philips tech. Rev.*, 1947–1948, vol. 9, no. 12, pp. 371–378.

If the periodically pulse-excited reverberant sound in a room is observed through an amplifier whose gain in decibels is a properly selected sawtooth function of time, the amplitude of the pattern on an oscilloscope can be made substantially constant, thus permitting the calculation of the reverberation time from the known rate of change of gain. This paper describes the circuits and techniques involved, and shows oscillograms depicting the effects of varying the gain change rate in simple and coupled rooms, and the effects of various pulse lengths of signal supplied to the loudspeaker. Vincent Salmon, USA

1184. **R. W. Young, "Example of propagation of underwater sound by bottom reflection,"** *J. acoust. Soc. Amer.*, July 1948, vol. 20, pp. 455–462.

The level of 200-cycle sound from an omnidirectional source 4.3 meters below the sea surface was measured with a hydrophone, usually 5, 15, or 90 meters deep. The ranges (between drifting and moving ships) were up to 80 km, over mud bottom. They were established by joint radio and acoustic signals. Sea depths were 3600, 1800, and 95 meters. Sample results are plotted as transmission anomaly (transmission loss other than that due to spherical divergence) versus range. Thermal structure of the ocean, sea state, and wind force are described.

At shorter ranges the sound pulses received without reflection from the bottom can be distinguished; their transmission anomaly follows rather well the theoretical curves which allow for surface reflection. At intermediate ranges only the bottom-reflected sound is recorded; at great distances direct and reflected pulses were not separated. Some features of the graphs are tentatively ascribed to interference effects after bottom reflection. The only conclusions from the examples cited are that sound at these frequencies can be propagated to great distances with the help of bottom reflection which over a considerable distance entirely nullifies spherical divergence. The observed transmission losses do not agree well with the commonly used empirical rules. A. O. Williams, Jr., USA

1185. L. N. Liebermann, "Reflection of underwater sound from the sea surface," *J. acoust. Soc. Amer.*, July 1948, vol. 20 pp. 498–503.

Measurements of the interference between direct underwater sound and that reflected from the surface of the sea were made at frequencies near 30 kc per sec. The sound source and hydrophone were mounted 10 ft below the surface and 150 ft apart, on towers which rested on the ocean bottom in 44 ft of water.

The author deduces that: (1) The state of the sea surface remains substantially unchanged for times of the order of 0.05 sec; (2) the amplitude of the reflected sound is frequently greater than that of the direct sound; and (3) there is no correlation between surface-wave height and amplitude of the reflected wave. This reviewer suggests that there should be some correlation between the duration of the state of the sea, the wave length of sound, and the time rate of change of the height of the sea.

Richard K. Cook, USA

1186. Charles J. Burton, "A study of ultrasonic velocity and absorption in liquid mixtures," *J. acoust. Soc. Amer.*, Mar. 1948, vol. 20, pp. 186–199.

The absorption and velocity of sound in liquid mixtures were measured in the frequency range 5-25 megacycles per sec. An improved optical-diffraction technique, with a multiplier-type phototube for measurement of the first-order diffraction spectrum, permitted rapid and precise measurements. The liquids were binary mixtures of water with alcohols, glycols, and glycol ethers. For a given mixture, there was no dispersion in velocity, and the sound absorption coefficient increased with the square of the frequency as expected. However, the velocity and absorption coefficient for some mixtures showed pronounced peaks at intermediate concentrations. For example, tertiary butanol and water had an absorption coefficient peak, more than 100 times that computed from the known viscosity and heat-conduction losses, at a 0.1 mol fraction concentration of the butanol. A velocity peak occured at a 0.05 mol fraction concentration for the same mixture.

The author speculates that the anomalous absorption is due to liquid crystal formation, or to intermolecular grouping between water and the other liquid, but as yet no really satisfactory theoretical explanation of the interesting experimental results has been developed.

Richard K. Cook, USA

1187. Harvey H. Hubbard and Arthur A. Regier, "Propeller-loudness charts for light airplanes," *Nat. adv. Comm. Aero. tech. Note*, no. 1358, July 1947, pp. 1–9.

A group of 57 graphs presents the calculated loudness levels for 2, 4, 6, and 8-bladed airplane impellers. Rotational noise, vortex noise, and total noise values are given for rotational tip Mach numbers ranging from 0.3 to 0.9. Propeller diameters of 6, 8, and 10 ft were considered at 50 to 200 mph with power inputs of 100 to 300 hp.

The work of L. Gutin and of Theodorsen and Regier formed the theoretical basis for the analysis. The experimental work [*Nat. adv. Comm. Aero. tech. Note*, no 1354] indicates close agreement of the calculations with theory for tip Mach numbers between 0.5 and 0.9, but for lower tip Mach numbers the actual sound levels are much greater than the calculated values. Results show that the loudness level decreases for a larger number of blades except at the lower Mach numbers where vortex noise tends to dominate rotational noise. This has some significance for fan design as well as airplane propeller design. A list of eight references is given.

Millard F. Dowell, USA

Elasticity Theory

(See also Revs. 1169, 1177, 1195, 1198, 1229)

1188. G. S. Shapiro, "Axially symmetrical deformations of an ellipsoid of revolution" (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, Dec. 1, 1947, vol. 58, pp. 1309–1312.

The author studies the stress distribution around a cavity in the shape of an ellipsoid of revolution due to hydrostatic tension at infinity. In an elliptic system of co-ordinates r, t, γ, the stresses σ_r, τ_{rt} are found to be

$$\sigma_r = -\frac{1}{r^2+t^2}\left[(r^2+1)\frac{\partial^2\Psi}{\partial r^2} + r\frac{\partial\Psi}{\partial r}\right] + \frac{1}{(r^2+t^2)^2}$$
$$\left[r(r^2+1)\frac{\partial\Psi}{\partial r} - t(1-t^2)\frac{\partial\Psi}{\partial t}\right] + \sigma\nabla^2\Psi + \frac{4(1-\sigma)}{r^2+t^2}(r^2+1)t\frac{\partial\Phi_1}{\partial r}$$

$$\frac{\tau_{rt}}{\sqrt{(r^2+1)(1-t^2)}} = \frac{1}{r^2+t^2}\frac{\partial^2\Psi}{\partial r\partial t} - \frac{1}{(r^2+t^2)^2}\left(r\frac{\partial\Psi}{\partial t} + t\frac{\partial\Psi}{\partial r}\right)$$
$$- \frac{2(1-\sigma)}{r^2+t^2}\left(t\frac{\partial\Phi_1}{\partial t} - r\frac{\partial\Phi_1}{\partial r}\right)$$

Here $\Psi = \Phi_0 + x\Phi_1$, where the harmonic functions Φ_0, Φ_1 can be expressed in the form of series

$$\Phi_0 = \sum_{n=0}^{\infty} A_n P_n(t) S_n(r), \quad \Phi_1 = \sum_{n=-1}^{\infty} C_n P_{n+1}(t) S_n(r),$$
$$S_n(r) = i^{-n} Q(ir)$$

σ represents Poisson's ratio and P_n the Legendre polynomials while A_n and C_n are constants.

The tensions σ_t, σ_γ on the surface of the ellipsoid $(t+0)$ are

$$\sigma_t = \frac{p\xi}{2N}\left[2(1+\sigma)c^2\xi - c(2\sigma\xi + 2\sigma + 7\xi) + 1 + 4\xi + 2\sigma\right]$$

$$\sigma_\gamma = \frac{p}{2N}\left\{\xi[2(1+\sigma)c^2\xi - c\left\{\xi + 6 + 4\sigma(2\xi - 1)\right\} + 4\sigma\xi + 3] + 2(1-\sigma)\right\}$$

In these equations

$$N = -(1+\sigma)c^2\xi^2 + c[\xi^2 - 2(1-\sigma)\xi] + \xi + 1 - \sigma$$

$$c = \frac{1}{\sqrt{\xi - 1}} \text{ arc } tg \sqrt{\xi - 1} \text{ for } \xi > 1, \quad c = \frac{1}{\sqrt{\xi - 1}}$$
$$\left[\ln(1 + \sqrt{1-\xi}) - {}^1/_2 \ln \xi\right] \quad \text{for} \quad \xi < 1$$

where $\xi = \dfrac{d}{\rho}$, $d = \sqrt{1 + r^2}$ is the semiaxis of the ellipsoid and ρ the radius of curvature at the end of d.

Z. Bažant, Czechoslovakia

1189. R. D. Mindlin, "Stress distribution around a hole near the edge of a plate under tension," *Proc. Soc. exp. Stress Anal.*, 1948, vol. 5, no. 2, pp. 56–68.

This paper presents a solution of the two-dimensional problem of a circular hole in a semi-infinite plate subject to tension parallel to the straight edge, the hole being located near the edge. Following Jeffery [*Trans. roy. Soc., Ser. A*, 1920, vol. 221, p. 265], a method involving the use of bipolar co-ordinates is employed. It is shown that there is an error in Jeffery's solution for this case

which leads erroneously to the prediction of a compressive stress at the minimum section on the straight boundary, in cases where the hole is close to the edge of the plate. The author's analysis shows that, contrary to Jeffery's results, this stress is always tensile but approaches zero as the hole approaches the edge of the plate. This situation is also confirmed by the photoelastic tests previously carried out by Brahtz, et al. [*Bureau of Reclamation tech. Memo.*, no. 597, Jan. 1940].

Stress-concentration factors, as found theoretically, are extremely high for cases where the hole is quite near the edge, that is, where the ratio c/R_2 approaches unity (c = distance from hole center to edge; R_2 = radius of hole). For example, where $c/R_2 = 1.02$ the concentration factor exceeds 10. The analysis shows that, for values of c/R_2 near unity, the stress along the straight boundary near the minimum section increases rapidly to a sharp peak a short distance away and then dies off. This rather peculiar stress distribution was confirmed by photoelastic tests carried out by the author for ratios $c/R_2 = 1.5$, 1.2, and 1.1. Although these tests were made on a plate of finite width while the analysis applies strictly only to a plate of infinite width, good qualitative agreement was obtained between theoretical and photoelastic stress-concentration factors and stress-distribution curves.

A. M. Wahl, USA

1190. F. Punga (with a discussion by W. Woebcken), "The direction of shear stresses (Die Richtung der Schubspannung)," *Wiss. Veröff. T. H. Darmstadt*, Jan. 1948, vol. 1, pp. 75–79.

The direction of the shear stress in a plane of given orientation through a given point is discussed with the aid of the Mohr sphere.

H. W. March, USA

Experimental Stress Analysis

1191. E. M. Saleme, "Three-dimensional photoelastic analysis by scattered light," *Proc. Soc. exp. Stress Anal.*, 1948, vol. 5, no. 2, pp. 49–55.

The author discusses scattering of light and the laws of artificial anistropy in three dimensions. He then applies these laws to three-dimensional photoelastic analysis and shows that the secondary principal stress difference in the plane of the wave front can be computed. As an example, the torsion problem is treated by the author showing that in a stress pattern obtained by scattered light viewed at 45 deg to the effective principal stress directions, the lines of constant phase difference are identical with the contour lines of the membrane analogy.

The author then describes the verification of this theory by experimental investigation of a semi-infinite block under concentrated axial load. He points out the incompleteness of his investigations, but shows that the results obtained are in good agreement with theory.

The reviewer considers that a concise introduction and summary, as well as a systematic list of notations would help the reader considerably. The paper lacks in these as well as in clarity and continuity.

Nicholas Sag, Australia

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 1178, 1198, 1201)

1192. H. D. Conway, "The large deflection of simply supported beams," *Phil. Mag.*, Dec. 1947, vol. 38, pp. 905–911.

In this paper the author derives an expression for the large deflection of a simply supported beam by taking into account the squares of the first derivatives in the moment equation. This problem is divided into two cases in which (*a*) the reactions at the ends of the beam are assumed to act vertically and (*b*) the reactions are assumed to act normally to the beam together with tangential frictional forces opposing deflections. The structure of the solution of case (*a*) is identical with that of the solution obtained for the large deflection of a cantilever beam by K. E. Bisshopp, D. C. Drucker, and the reviewer in the *Quarterly of Applied Mathematics*, 1945, vol. 3, the only difference being that the cantilever beam deflects less than the approximate deflection $\delta = PL^3/3EI$, while the simply supported beam deflects more than the approximate deflection $\delta = PL^3/48EI$.

When the reactions are assumed normal to the beam, it is shown that there is a lateral component tending to cause a greater deflection, this being partially resisted by frictional forces. With an increase of deflection and slope, the buckling component of the normal reaction becomes predominant.

Henry J. Barten, USA

1193. C. Casci, "Theoretical research on stresses in the eye of a connecting rod (Ricerche teoriche sulle tensioni dell'occhio di biella)," *Aerotecnica*, Apr. 1948, vol. 28, pp. 67–73.

The stresses in the eye of a connecting rod are computed from curved-beam theory on the assumption that the wrist pin applies a distributed radial load to the eye, which can be described by $p_0\cos^2\theta$ where θ is the angular distance from the vertex of the eye. The unknown bending moment and axial force transmitted at the vertex are obtained by a conventional least-work analysis.

Numerical solutions are given for the distribution of bending moment and axial force along eyes having three different sections, a rectangular section, a section with two fins, and a section with four fins. The computed bending moments and forces are compared with those for concentrated loads. It is found that a distribution of the load in proportion to $\cos^2\theta$ reduces the maximum bending moment to about one fifth; the maximum value of the axial force remains about the same, but its distribution is changed radically.

Walter Ramberg, USA

Plates, Disks, Shells, Membranes

(See also Revs. 1196, 1200, 1201)

1194. K. G. Tester, "Contribution to the calculation of the hyperbolic paraboloid shell (Beitrag zur Berechnung der hyperbolischen Paraboloidschale)," *Ingen.-Arch*, vol. 16, no. 1, pp. 39–44.

The author studies the horizontal components of stress produced in a membrane $z = kxy$ by a horizontal wind directed along the y-axis. Similar stress determinations are made for various types of hydrostatic and wind loads, varying linearly and quadratically in any direction.

D. L. Holl, USA

1195. E. Volterra, "On the general problem of a plate supported by elastic soil (Sul problema generale della piastra poggiata su suolo elastico)," *R. C. Accad. Lincei*, May 1947, vol. 2, pp. 596–598.

By using the Fourier transform, the problem of an infinite plate of constant thickness lying on an elastic base is treated in the case that the Green's function for the base is known. The formulas derived are applied to a plate subject to a rectangular load only, with the assumption of either the hypothesis of Winkler, or that of Jodl, for the Green's function of the base.

F. A. Willers, Germany

Buckling Problems

(See also Revs. 1192, 1200, 1206)

1196. A. van der Neut, W. K. G. Floor, and I. Binkhorst, "Experimental investigation of the postbuckling behavior of stiffened, flat, rectangular plates under combined shear and compression. Part I" (in English), *Nat. LuchtLab. Amsterdam Rap.*, no. S.300, Apr. 1947, pp. 1–57.

This is a report on the first part of an experimental investigation of the diagonal tension field in stiffened flat plates loaded by a combination of shear and of compression parallel to the shorter edges of the individual panels. The five individual panels had a length-to-width ratio of 3.16; the ratio of plate thickness to panel width was $^1/_{167}$; results for other ratios will be presented subsequently. The longitudinal edges of the individual panels were approximately simply supported. Special strain adjusters were used to distribute the shearing load over the end stiffeners, and to equalize the compressive strains in the longitudinal stiffeners.

In the tests the ratios of external compression to shear, of load to buckling load, and of stiffness to compression strain of the longitudinal stiffeners were varied. Measurements included the longitudinal and lateral strains, shearing strain in the panels, and longitudinal stiffener loads. The shape of the buckled plate was determined by means of profile and slope recordings. Stress distributions in the plate were calculated from the deformation measurements.

Results are presented in tables and curves. Wave-form ratios are introduced to indicate the degree of development of the tension field as compared with the Wagner tension field. Based on these test results a preliminary proposal for practical applications is made. Marked differences in deformations, stiffener loads, and panel loads were found between inner and outer panels, indicating that an insufficient number of panels was used.

P. S. Symonds, USA

1197. Ch. Massonnet, "Buckling of bars of open cross section and thin walls (Le flambage des barres à section ouverte et à parios minces)," *Annir. Vol. Univ. Liège*, 1947, pp. 1–16.

The purpose of this paper is threefold: (1) To summarize the existing theory of combined flexural and torsional buckling; (2) to add a new contribution to this theory; and (3) to attempt a synthesis of local buckling of the walls and flexural and torsional buckling.

The first point is treated rather briefly, the reader being referred to a paper by S. Timoshenko [*J. Franklin Inst.*, 1945, vol. 239, nos. 3,4,5]. The author's treatment differs by deriving the basic equations by minimizing the difference between the increase in internal potential energy and the work of the external forces during buckling. The solution is given for a bar simply supported at both ends and subjected to eccentric compression. The effect of a shear force at the end of the bar is also studied; it is found, for example, that the critical load can be determined, to a first approximation, by replacing the linearly varying bending moments by their mean value.

Buckling in torsion and flexure combined with local failure of the walls is also studied by means of the energy principle. The case of a uniformly compressed T-section is worked out in detail, and it is found that the effects of local wall buckling on the critical load are very small.

The author briefly discusses the effect of buckling beyond the elastic limit, proposing that von Kármán's reduced modulus be used at appropriate places instead of Young's modulus. Recent investigations [F. R. Shanley, *J. aero. Sci.*, May 1947, vol. 14, p. 261] show that these remarks must be interpreted cautiously.

G. H. Handelman, USA

Joints and Joining Methods

(See also Revs. 1218, 1220)

1198. F. Stüssi, "Composite beams (Zusammengesetzte Vollwandträger)" (with summaries in French and English), *Publ. int. Ass. Bridge Struct. Engng.*, 1947, vol. 8, pp. 249–269.

The author studies the distribution of internal forces in composite beams. He shows that magnitudes and directions of the stresses depend on the stiffness and arrangement of the joining means employed. The problems are solved by the usual method for statically indeterminate systems with the help of the stress-strain relations. Several actual examples taken from wooden, steel, and concrete structures are calculated. Some of the common time-saving rules, often used by the structural engineer, are critically discussed and shown to be untenable.

Ragnar Nilson, Sweden

Structures

(See also Revs. 1198, 1206, 1223, 1226)

1199. G. Vedeler, "The practical calculation of intersecting girders," *Shipbuilder*, July 1948, vol. 55, pp. 435–444.

Numerous formulas are presented, mostly without derivation, for the deflections and moments in intersecting girders as used in ship construction. The torsional stiffness of the members is neglected. The gridworks that are considered have simplified plans, or structural layouts, which permit the development of analytical formulas for deflections and bending moments. The loading is considered to be uniform longitudinally, and uniform or triangular in the transverse direction. Formulas are given for 1, 2, or 3 longitudinal girders with any number of transverse members (stiffeners). The stiffeners are of equal length and generally of the same stiffness.

A design method is proposed in which, by a proper choice of the stiffness of the members, the maximum negative moment in the stiffener nearest the end of the girders is made equal to the maximum positive moment in the stiffener at the center of the girder span. Graphs and formulas are given for use in this design procedure. The proposed design method appears to offer considerable advantage over the current practice in which structural deflections are disregarded in calculating moments in the girders.

Stanley U. Benscoter, USA

1200. A. H. Stang and M. Greenspan, "Perforated cover plates for steel columns: summary of compressive properties," *J. Res. nat. Bur. Stands.*, May 1948, vol. 40, pp. 347–359.

This paper summarizes the results of tests to determine the mechanical properties of perforated cover plates to be used in built-up boxtype columns. The experimental axial stiffnesses of 86 of such columns, determined in the elastic range, are compared with theoretical values previously derived by Greenspan [*J. Res. nat. Bur. Stands.*, 1943, vol. 31, p. 305, and 1946, vol. 37, p. 157]. In general, the agreement is very good (within a few per cent).

The measured stresses on the edges of the perforations, which include circular, elliptical, square, and ovaloid shapes, are also in good agreement with theory.

Maximum compressive load tests show that the net area of the perforated plate columns may safely be used for estimating the strength of columns with perforated cover plates.

S. B. Batdorf, USA

1201. **W. Olszak, "On the theory of prestressed beams and plates (Z teorii belek i płyt wstępnie sprežonych)," *Inžyn. Budown.*, Feb. 1947, vol. 4, pp. 94–112.**

This is an introduction to the design of prestressed beams. The dimensioning of the wire and its optimum location in the beam are discussed, and the pertinent formulas derived. Transverse stresses and "prebending" are briefly treated.

A. W. Wundheiler, USA

Plastic Flow, Failure; Mechanics of Solid State

See also Revs. 1181, 1211, 1214, 1221, 1222, 1229, 1293)

1202. **W. T. Lankford, J. R. Low, and M. Gensamer, "The plastic flow of aluminum-alloy sheet under combined loads,"** *Trans. Amer. Inst. min. metall. Engrs.*, 1947, vol. 171, pp. 574–604.

The main object of this study was to measure the ductility of 0.040-in. aluminum-alloy sheets under biaxial stresses. A hydraulic bulge test was adopted, using circular bulges to obtain equal biaxial stresses and elliptical ones for other stress ratios. It is shown from a comparison of stress-strain curves obtained in the bulge tests with those from other tests that there is little size effect, and that the concept of a "significant" ("generalized") stress-strain curve is approximately valid within the range of homogeneous strain. It is emphasized that the bulge test permits a range of homogeneous strain to be achieved which is at least twice that in an ordinary tensile test.

On the basis of this significant stress-strain curve, the Lévy-Mises plastic stress-strain relations, and the assumption that in state of biaxial tension rupture occurs when the greatest tension reaches a critical value, the authors obtain a simple theoretical formula for the ductility under biaxial loading of those alloys in which rupture precedes instability. The available experimental checks of this formula are few but are in fair agreement considering the anisotropy of rolled sheet. Observations of the types of fracture are also presented.

Rodney Hill, England

1203. **W. Prager, "Problem types in the theory of perfectly plastic materials,"** *J. aero. Sci.*, June 1948, vol. 15, pp. 337–341.

The author's term "perfectly plastic" refers to materials which do not exhibit strain hardening. In a redundant structure some portions may reach the yield stress before the other members, and, under further loading, deform without increase in stress ("contained plastic deformation"). Finally all portions reach the plastic range ("unrestricted plastic flow").

The present paper discusses various principles of minimum strain energy, as applied to the intermediate case of contained plastic deformation of such materials, including unloading. A simple example is used and the principles are illustrated graphically. In the case of repeated loading the system may "shake down" to a state of residual stress such that it behaves in an elastic manner under further cyclic loading within a certain range. A tentative strain-energy theorem for this situation is suggested. By means of the same example the problem of structural stability in the plastic range is shown to involve the history of deformation.

The discussion applies in a qualitative manner to "strain-hardening" materials, since the over-all behavior of the redundant structure exhibits a sort of strain hardening even though the materials of which it is composed do not. To this reviewer, this suggests reversing the usual concept of strain hardening by considering, instead, "stress softening" of individual elements under conditions resembling those of contained plastic deformation.

F. R. Shanley, USA

1204. **A. B. Bagsar, "Development of cleavage fractures in mild steels,"** *Weld. Res. Suppl.*, Mar. 1948, vol. 27, pp. 97–123.

The author describes a new test for the determination of the tensile breaking loads of edge-notched rectangular steel sections and examines the conditions under which failure occurs by cleavage fracturing. The dependence of the results on the different properties of the specimen are studied. Among these are eccentricity of loading, section depth and thickness, notch geometry, temperature of testing, rate of loading, heat treatments, direction of rolling.

There are two distinct modes of failure of steel, one being by cleavage or brittle fracturing, the other by shear fracturing preceded by plastic flow. In the first case the breaking strength was found to be nearly the same for steels of the same nominal tensile strength, and lower than the breaking strength by shear fracturing. Many interesting microphotographs are given, showing the various modes of dependence of cleavage fracture on the deformation and distortion of the crystal grains of the different materials.

Th. Pöschl, Germany

1205. **A. B. Bagsar, "Cleavage fracturing and transition temperatures of mild steels,"** *Weld. Res. Suppl.*, Mar. 1948, vol. 27, pp. 123–131.

In continuation of the foregoing paper, test details and data on determination of transition temperatures and on behavior of steels with respect to these temperatures are given. Transition temperature can be defined as the temperature at which the mode of fracturing changes from the cleavage or brittle type to the shear or ductile type. The paper further gives some equations of semiempirical character relating the different quantities investigated, for instance

$$P/A = (P/A_0) + a(T - T_0)^b$$

where P/A is the unit breaking load at temperature T, P/A the same at the point of inflection (that is, the theoretical transition temperature T_0), and a and b are constants for given steel and test conditions.

Within certain limits the transition temperatures determined by the cleavage tear test can be correlated with those by the conventional Charpy impact test. The transition temperatures determined by these tests can also be correlated with service conditions.

Th. Pöschl, Germany

1206. **P. P. Bijlaard, "Some contributions to the theory of elastic and plastic stability"** (in English with summaries in French and German), *Publ. int. Ass. Bridge Engng.*, 1947, vol. 8, pp. 17–80.

In the light of the theory of elastic and plastic stability, it is shown that the results of Kollbrunner's experiments [*Inst. Baustat. T. H. Zurich*, no. 17, 1946] on the buckling of plates in the plastic range show a striking agreement with the author's theory of plastic stability of plates of steel and aluminum. The basic point is that, with the buckling of plates and shells, another state of stress (with another ratio of the deviator components) is superposed on the initial state, and, with the changing ratio of the deviator components, the material no longer behaves quasi-isotropically, but assumes anisotropic characteristics.

The theory is applied to several cases of buckling of plates, and is also extended to cover webs of bridges and girders, T-stiffeners, flange angles, latticed struts, timber columns, and sandwich plates. The stability of wharves, bridge piers, and of portals of a through-truss bridge are considered in detail on the basis of Haarman's method of virtual buckling length.

S. K. Ghaswala, India

1207. J. G. Oldroyd, "Two-dimensional plastic flow of a Bingham solid: a plastic boundary-layer theory for slow motion," *Proc. Camb. phil. Soc.*, July 1947, vol. 43, pp. 383–395.

By a Bingham solid is meant a body that behaves like an elastic one as long as the components p'_{ik} of the stress deviator satisfy the condition

$$\tau^2 = \frac{1}{2}\sum_{i,k} p'_{ik}{}^2 < \vartheta^2$$

and, for $\tau < \vartheta$, like an incompressible viscous fluid with the viscosity coefficient η depending on the stresses

$$\eta = \frac{\eta_1}{1 - \vartheta/\tau}$$

For $\vartheta = 0$, $\eta = \eta_1$ this becomes the ordinary viscous fluid; for $\eta_1 = 0$, $\tau = \vartheta$ it is the ideal plastic body with elliptic plasticity limit. Dynamic similarity depends on two dimensionless parameters, the Reynolds number R_1 (formed with η_1) and

$$S = \frac{\vartheta}{\eta_1 V/D}$$

where V is a typical velocity and D a typical length.

In the present paper the fourth-order, nonlinear differential equation for the stream function ψ of a two-dimensional flow is set up, and then the boundary-layer equation, for large S and R_1 of the order 1, is derived. Three particular types of solutions are discussed. The first concerns the flow along an infinite rigid wall, and represents an exact solution (for any R_1 and S) of the complete theory

$$u = \frac{Vy}{\delta}\left(2 - \frac{y}{\delta}\right),\ v = 0,\ dp/dx = -\frac{2V\eta_1}{\delta^2}\ (y \leqq \delta),\ \delta = \text{const.}$$

The second example, with a variable layer thickness δ, but constant pressure along $y = \delta(x)$, applies to the flow on both sides of an infinitesimally thin blade that penetrates into the body at velocity V. Here, the flow component parallel to the blade is given by

$$u = -V(1 + 2y/\delta)(1 - y/\delta)^2$$

with δ proportional $x^{-2/3}$. A similarly simple expression is finally found for a boundary layer "remote from rigid boundaries" which corresponds to a "jet" of Bingham material penetrating into an elastic body. R. von Mises, USA

1208. J. G. Oldroyd, "Rectilinear plastic flow of a Bingham solid: I. Flow between eccentric circular cylinders in relative motion. II. Flow between confocal elliptic cylinders in relative motion. III. A more general discussion of steady flow. IV. Nonsteady motion," *Proc. Camb. phil. Soc.*, 1947, vol. 43, July, pp. 396–405; Oct., pp. 521–532; 1948, vol. 44, Apr., pp. 200–213 and pp. 214–228.

The first three papers deal, in an exhaustive way, with the following case of motion of a Bingham body (see review no. 1207): All particles move parallel to the z-axis at constant velocities $w(x, y)$ under an over-all uniform pressure p; the flow is enforced by two cylindrical surfaces, one at rest and one moved at velocity V in the z-direction, enclosing between them the Bingham material. According to the basic conditions stated in the quoted review, the function $w(x, y)$ is subject to the second-order nonlinear differential equation

$$\text{div}\ \eta_1\left[\text{grad}\ w + \vartheta\frac{\text{grad}\ w}{|\text{grad}\ w|}\right] = 0 \ldots\ldots\ldots\ [*]$$

with $w = 0$ and $w = V$ respectively at the two contours. The solution depends on the shape of the contours and on one dimensionless parameter $S = \vartheta D/\eta V$ where D is some linear dimension.

If the two contours are concentric circles, the solution can be given immediately in finite form. In the case of eccentric circles and, later, of confocal ellipses, the author uses the complex variable $z = x + iy$ for which the differential equation takes the form

$$\frac{4\eta_1}{\vartheta}\frac{\partial^2 w}{\partial z \partial \bar{z}} + \frac{\partial}{\partial \bar{z}}\left[\frac{\partial w}{\partial z}\left(\frac{\partial w}{\partial z}\frac{\partial w}{\partial \bar{z}}\right)^{-1/2}\right] + \frac{\partial}{\partial z}\left[\frac{\partial w}{\partial \bar{z}}\left(\frac{\partial w}{\partial z}\frac{\partial w}{\partial \bar{z}}\right)^{-1/2}\right] = 0$$

An approximate solution can be found by developing

$$w = V\,[w_1 + Sw_2 + S^2w_3 + \ldots]$$

where w_1 obviously represents the velocity distribution for an ordinary viscous fluid. The author determines w_2 and, in the second case, also w_3 by using certain developments in powers of the co-ordinates.

In the third paper a general solution of the inverse problem is presented. The author introduces a "natural" co-ordinate system w, W, where W = const are supposed to be the orthogonal trajectories to the lines of constant velocity w. If the factors h_1 and k_1 in the formula for the length element

$$ds^2 = h_1{}^2dw^2 + k_1{}^2dW^2$$

are known as functions of w, W, a definite velocity distribution will be defined. The author shows that by setting $h_1 = 2\eta_1/\vartheta(\xi - 1)$, $k_1 = 2\eta_1/\vartheta(\xi + 1)$, one can use for $\xi\,(w, W)$ any solution of a certain linear differential equation of second order. Some examples are given. As a general conclusion may be mentioned the appearance of a region of elastic solid (w = const) when S exceeds a critical value S_o, and the excessive increase in the gradient of velocity in some part of the region (boundary layer) for highly increasing S.

In the fourth paper a certain type of nonsteady flow is discussed. Here, w is a function of x, y, t and the pressure gradient, having the z-direction, is supposed to be a given function $P(t)$. Then, the right hand side of Equation [*] becomes $P(t) + \rho\partial w/\partial t$. The analogy with the problem of heat conduction is discussed. Explicit solutions are given for the flow between parallel plane boundaries. In particular, it is shown how the velocity distribution in the neighborhood of a moving plane varies with its acceleration. Finally, the initial velocity distribution of plastic material in a circular pipe, suddenly subjected to a pressure gradient, is calculated. R. von Mises, USA

1209. A. P. Sokolov, "On the elastic-plastic state of a membrane" (in Russian), *Notes Acad. Sci. USSR* (*Doklady Akad. Nauk SSSR*), Apr. 1, 1948, vol. 60, pp. 33–36.

This is a theoretical study of the following problem: An infinite thin plate with a circular hole is stretched by principal stresses whose values at a great distance from the hole are

$$\sigma_x = q\sigma_s \qquad \text{and} \qquad \sigma_y = p\sigma_s$$

where σ_s denotes the stress at the yield point determined by a tension test, and $0 < q \leq 1$, $0 < p \leq 1$.

Assuming the Saint Venant yield condition (condition of plasticity), the author takes advantage of the solution by V. V. Sokolovski in his "Theory of plasticity" (in Russian) to determine the boundary of the plastic zone surrounding the hole, and gives the equation of this boundary in polar co-ordinates.

M. T. Huber, Poland

1210. **E. G. Thomsen, I. Cornet, I. Lotze, and J. E. Dorn, "Investigation on the validity of an ideal theory of elastoplasticity for wrought aluminum alloys,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1552, July 1948, pp. 1–47.

This report gives the relation between stresses and plastic strains for wrought aluminum and magnesium alloys subjected to tension, compression, and torsion loadings. It was found that the theory was not suitable for predicting the stress-strain curves for the wrought aluminum alloys. Fair agreement between tests and theory was found for a cast and solution heat-treated R magnesium alloy. For some of the aluminum alloys, and for the compression and torsion data, correlation between test results was obtained when shear stresses were plotted as functions of effective strain. However, this correlation has no theoretical justification.

It is pointed out that the failure of the theory may be due to anisotropy of plastic deformation, discontinuity of the structure, density variations with deformation, and the assumption of linearity between stress and infinitesimal strains.

Joseph Marin, USA

1211. **W. Boas and M. E. Hargreaves, "On the inhomogeneity of plastic deformation in the crystals of an aggregate,"** *Proc. roy. Soc. Lond. Ser. A*, Apr. 22, 1948, vol. 193, pp. 89–97.

A study of the variation of plastic deformation in large aluminum crystals in a polycrystalline aggregate is reported. Hardness and local elongation at various points were determined for thin bars subjected to tensile loads. Variation of plastic deformation from grain to grain was noted, and, in addition, variation within each grain was found, such that as the grain boundary was approached for a particular grain the deformation approached that of its neighbor. Further, the results indicated a nonuniform distribution of stress across the test specimen. The difficulties of obtaining stress-strain curves for polycrystalline aggregates from single crystal data are discussed, and a mechanism for deformation other than by slip is suggested for polycrystalline structures in the vicinity of grain boundaries.

Louis F. Coffin, Jr., USA

1212. **C. W. MacGregor and N. Grossman, "Some new aspects of the fatigue of metals brought out by brittle transition temperature tests,"** *Weld. Res. Suppl.*, Mar. 1948, vol. 27, pp. 132–143.

For an SAE 1020 steel a series of experiments are described in which the effects of different numbers of prior cycles of fatigue stress on the brittle transition temperature and on the brittle fracture strength are determined. The most important results of these tests are the following:

1 The brittle transition temperature was increased greatly and continuously as the number of prior fatigue cycles increased at stresses both above and slightly below the endurance limit.

2 The brittle fracture strength was greatly and continuously decreased as the number of prior cycles of fatigue increased at all of the stress levels investigated.

3 An analytical relation between the number of prior cycles of fatigue and the brittle transition temperature was formulated and represented by graphs.

4 The effect of fatigue cycles on the brittle fracture properties shows that they depend only on the past history of stressing and on the instantaneous values of strain, strain rate, and temperature.

Several microphotographs and an extended bibliography of the whole subject are given. Th. Pöschl, Germany

1213. **A. I. Gubanov, "On the theory of structured liquids"** (in Russian), *J. tech. Phys.* (*Zh. tekh. Fiz.*), 1947, vol. 17, no. 6, pp. 629–640.

An elastic-viscous material, characterized by two shear moduli G_1, G_2, and two viscosities η_1, η_2 (a "structured" material) can be characterized by the operator

$$\frac{\eta_1 G_2(1 + \tau_2\partial/\partial t)\partial/\partial t}{\eta_1\partial/\partial t + G_2(1 + \tau_1\partial/\partial t)(1 + \tau_2\partial/\partial t)} \cdots\cdots\cdots [*]$$

where $\tau_1 = \eta_1/G_1$, $\tau_2 = \eta_2/G_2$. If the shear modulus G in the equations of elasticity theory is replaced by the operator [*], the theory of structured materials is obtained. This procedure is used to determine the elementary longitudinal and transverse waves, and the results are interpreted. The problem of a sheet adhering to a layer of a structured material is solved by verification; the one of relaxation vibrations of such a sheet is solved by means of Laplace transforms. The results are used to explain two of a series of Russian experiments with jellylike liquids some of which demonstrated shape "memory." (Similar phenomena have been observed with plastics in the United States.)

A. W. Wundheiler, USA

Design Factors, Meaning of Material Tests

1214. **G. C. Noll and M. A. Erickson, "Allowable stresses for steel members of finite life,"** *Proc. Soc. exp. Stress Anal.*, 1948, vol. 5, no. 2, pp. 132–143.

Allowable stresses are usually established on the basis of endurance limits for infinite life, that is, for fluctuating loads applied more than one million times. But many members, working under ordinary load fluctuations, have to resist higher overloads for less than one million times. In this case the piece may be considered as having a "finite life," and consequently the allowable stress may be increased.

In this paper indications for establishing the allowable stresses for steel members having a finite life are given. Taking into account fatigue tests for fluctuating loads, stress variations, surface finish, and stress concentration, the authors establish a general fatigue diagram and, on the basis of data covering 18 hardness ranges of common structural steel, they present 18 diagrams which permit the selection of allowable stresses for fluctuating overloads applied a limited number of times.

Aurel A. Beles, Roumania

Material Test Techniques

(See also Revs. 1196, 1204, 1205, 1211, 1220)

1215. **J. Winson, "The testing of rotors for fatigue life,"** *J. aero. Sci.*, July 1948, vol. 15, pp. 392–402.

This paper presents the theoretical basis for a proposed experimental method for fatigue testing of articulated rotor blades such as are used in rotating wing aircraft. The equations for blade loading in flight are first derived. Equations are then derived for the stresses in a rotating wing subjected to a second harmonic control (pitch variation at twice rotor speed). It is concluded that the steady and alternating stresses of flight at specified sections can be closely approximated by a suitable whirl test in which the rotating speed, collective pitch, second harmonic control, and damper at the drag hinge are independently specified.

A sample calculation is presented to determine the test parameters for a rotor of given physical characteristics and specified bending moment ranges at the inboard and mid-sections. The path which experimental work should take is indicated, but no test data are presented. S. S. Manson, USA

**1216. P. Chevenard, "The micromechanic testing of metals (L'essai micromécanique des métaux)," ** *Schweiz. Verb. MatPrüf. Tech. Ber.*, no. 42, Apr. 1947, pp. 3–36.

The article consists of two parts, the first dealing with the construction of various apparatuses used for micromechanic testing of materials, the second giving a number of applications.

The author gives an explanation of the choice of the test bar dimensions (diameter 1.5 mm and length 7.5 to 10 mm). The following microtesting machines, mostly commercially available, are described: (1) for tension, shear, and bending; (2) for tension of fine wires, textile-fibers, etc.; (3) for continuous torsion; (4) for alternating torsion; (5) for the measurement of creep at high temperatures, and (6) for impact tests.

The necessary amplifications of the deformation and the magnitude of the force are obtained optically, and the displacements of the light-spot are registered on a photographic plate. It is claimed that the accuracy is very good. The subjects in the second part have the following headings: (1) Real mechanical properties of materials. Deviations from Hooke's Law. (2) Plasticity of metal crystals and microcrystalline yielding. (3) Crystalline anisotropy of metals due to deformation at low temperature. Choice of the Poisson ratio as a proof of anisotropy. (4) Micromechanical study of a heterogeneous piece of material. (5) Mechanical hysteresis and fatigue.

In each part valuable remarks are made, and the results presented in the form of a great number of graphs and pictures.

R. G. Boiten, Holland

1217. J. W. Ballou, "A stress-strain tester for textiles employing a magnetic strain gage," *Amer. Soc. Test. Mat. Bull.*, no. 150, Jan. 1948, pp. 38–42.

The tensile stress-strain properties of textile fibers under controlled rates of elongation may be investigated by means of the testing apparatus described in this paper. The applied load is measured with the aid of a General Electric magnetic strain gage which is employed to give the deflection of a stiff cantilever beam to which the upper end of the fiber is attached. Constant adjustable elongation rates are maintained by use of a G-E "Thymo-trol" drive geared to a worm-threaded nut and shaft arrangement to which the lower end of the specimen is attached. Recording is performed by means of a recording chart potentiometer, the strain gage signal having been amplified, rectified, and recorded as one co-ordinate. Time which is proportional to fiber elongation is used as the other co-ordinate.

Various details of the machine are described and sample records are included. Testing velocities range from 0.04 to 18 in. per min.

Louis F. Coffin, Jr., USA

1218. E. Thomas, "Radiology of joints in welded piping for power plants," *Proc. Instn. mech. Engrs.*, 1948, vol. 158, no. 1, pp. 1–8.

This paper describes apparatus and procedures in use in England for x-ray and gamma-ray examination of joints in welded piping in power plants. Details of portable equipment, film and film supports, penetrameters, exposure times and methods are given for a variety of pipe sizes, locations, and types of faults to be detected. Difficulties in locating source and film on pipe in plants with limited clearance are described and suggestions advanced for meeting them. Double exposure and overlapping film procedures for localizing faults are described. Gamma-ray source offers convenience in use but results are not as satisfactory as x ray. Many suggestions are made for improvements in apparatus. The paper is a practical statement of the problems encountered in the author's experience.

Henry A. Lepper, Jr., USA

1219. P. Le Rolland, "On a new method of determining the energy dissipation due to internal friction in solid bodies (Sur une nouvelle méthode de détermination de la dissipation d'energie par frottement interne dans les corps solides)," *C. R. Acad. Sci. Paris*, July 5, 1948, vol. 227, pp. 37–39.

Evaluation of a damping constant for engineering materials usually proceeds from observations of the decrease in amplitude of torsional vibrations. The author proposes a method in which a member of the material is clamped at one end, and at the other end supports a frame to which two identical gravitational pendulums are attached. If the pendulums are started vibrating 180 deg out of phase, the decrease in the amplitude of vibration of the pendulums is due to air friction and friction at the supports. If the pendulums are started in phase, the decrease in amplitude includes in addition the effect of the internal damping of the supporting member. A measure of the internal damping may be developed from the decreases in amplitude under the two starting conditions, and the frequency of beats resulting if the pendulums are started 90 deg out of phase.

Glenn Murphy, USA

Mechanical Properties of Specific Materials

(See also Revs. 1204, 1205, 1210, 1211, 1212, 1217, 1220, 1225, 1230)

1220. A. R. C. Markl, "Fatigue tests of welding elbows and comparable double-miter bends," *Weld. Res. Suppl.*, June 1948, vol. 13, pp. 310–320.

Full-scale fatigue data are presented on 90-deg welding elbows and on comparable double-miter 90-deg elbows, in relation to the code for pressure piping. Both types of elbow were tested in reversed bending, not only in the plane of curvature, but also normal to this plane. Failure was considered to occur when water, which filled every specimen, appeared on the outside. The direction and location of typical fatigue cracks agree well with theory.

The results are presented as curves of nominal combined stress versus number of cycles to failure. These curves show that a welding elbow can sustain about 2.4 times as many stress reversals in the plane of bending, and about 13.5 times as many transverse to this plane, as can a miter elbow. In the discussion, Messrs. Pardue and Vigness present experimental curves of longitudinal and transverse stress as functions of position around the circumference of a 90-deg tube-turn, and compare them favorably with theory.

B. G. Rightmire, USA

1221. A. E. Flannigan, L. F. Tedsen, and J. E. Dorn, "Stress rupture and creep tests on aluminum-alloy sheet at elevated temperatures," *Trans. Amer. Inst. min. metall. Engrs.*, 1947, vol. 171, pp. 213–245.

Specimens of five clad aluminum alloys, in the form of 0.040-in. sheets, were loaded in tension in the cross-grain direction at testing temperatures of 94 F, 211 F, 300 F, and 375 F. Data are presented for rupture stress for times up to 1000 hr, elongation at fracture, and creep rate. In many cases the elongation at fracture in stress-rupture tests was less than that obtained in short tensile tests. Rupture stresses at temperatures above 150 F were generally lower than values of yield stress determined in short-time tensile tests. On the basis of rupture stress in 1000-hr tests the materials ranked in different order at temperatures above 211 F, as compared with tests at temperatures below 211 F.

T. J. Dolan, USA

Mechanics of Forming and Cutting Processes

(See also Rev. 1202)

1222. **S. I. Goobkin, "Theory of flow stresses in metals at drawing"** (in Russian), *Bull. Acad. Sci. USSR Ser. tech. Sci.* (*Izv. Akad. Nauk SSSR Ser. tekh. Nauk*), Dec. 1947, no. 12, pp. 1663–1681.

The author analyzes critically several existing formulas expressing the flow stress k in terms of the ultimate strength, the angle α characterizing the drawing, the coefficient of external friction μ, and the reduction per draw. He regards the formula of Körber (1940) as the best one, but points out that it does not take into account all the "components" of the resistance of drawing, and can therefore be applied to drawing solid bars, but not tubes.

The author derives new formulas for the stress from the assumption that, in a first approximation, this stress is a sum of three "components" k_1, k_2, and k_3 which, in accordance with simplified technological methods, are defined in a different manner from that used in the mathematical theory of plasticity. Also a new formula for the reduction per draw and a new determination of external friction are given.

The figures are burdened by conspicuous errors, and obscure. The formulas deduced are compared with the author's and other experimental results, and improved by means of empirical coefficients when necessary. M. T. Huber, Poland

Soil Mechanics; Seepage

(See also Rev. 1293)

1223. **F. B. Nelson-Skornyakov, "Filtration through dams and dikes with a core or screen (impervious foundation)"** (in Russian), *Notes Acad. Sci. USSR* (*Doklady Akad. Nauk SSSR*), June 1, 1948, vol. 60, pp. 1137–1140.

The paper presents a rigorous solution of the problem of filtration through a homogeneous core of a dam in the shape of an oblique triangle resting on an impervious horizontal foundation, when the upstream water level coincides with the vertex of the triangle. The method employed is that of conformal representation. Expressions for the equipotential lines, flow lines, and discharge are stated in terms of complicated infinite series. Numerical values of discharge for some special cases are also given.

Alexander Hrennikoff, Canada

(Editor's Note: A six page translation of this paper has been made by Dr. Hrennikoff and the editor can supply photostats at the usual rate.)

1224. **H. E. Babbitt and D. H. Caldwell, "The free surface around, and interference between, gravity wells,"** *Univ. Ill. engng. Exp. Sta. Bull.*, no. 374, Jan. 7, 1948, vol. 45, pp. 9–58.

The electrical analogy and sand models were used to investigate the flow rate into, and the shape of the free surface of the ground water around a single well, the effects on the flow rate of interference among several adjacent wells, and those of partial penetration of a single well into the water-bearing stratum. The results show that Dupuit's equation yields correct values for the flow rate Q, but does not take account of the seepage area above the water surface in the well. The authors develop another equation for the free surface.

Muskat's equations for interference between adjacent wells were checked experimentally for several cases.

Joseph Levy, USA

1225. **A. W. Skempton, "A study of the geotechnical properties of some postglacial clays,"** *Géotechnique Lond.*, June 1948, vol. 1, pp. 7–22.

The paper considers two clays from southern England and three from northeastern United States. The effect of recession of the ice cap on the processes of deposition and loading of these clays is considered. Casagrande's method of determining preconsolidation load is assumed valid, and some instructive relationships between preconsolidation load, shear strength, geologic history, and water content are obtained which seem to justify the assumption to some degree. Understanding of the shearing properties of clays is aided by definition and illustration of the concepts of true cohesion and angle of internal friction, of effective cohesion and angle of shearing resistance, and of angle of shearing resistance in terms of preconsolidation load; the triaxial shear test at constant water content, at zero neutral pressure, and at preconsolidation pressure is used as a basis for data and argument. The $\phi = 0$ analysis of stability is described. It is concluded that all of the clays possess considerable internal friction, and that the three American clays possess little or no true cohesion.

C. Martin Duke, USA

1226. **ASCE Subcommittee on Slope Protection, "Review of slope protection methods,"** *Proc. Amer. Soc. civ. Engrs.*, June 1948, vol. 74, pp. 845–866.

This report is a summary of available information.

A procedure based on semiempirical formulas enables the designer to compute wave heights, lengths, and velocities, knowing the speed of wind and the fetch. A formula for the wind setup is also included. The minimum size of stones that cannot be moved by the waves can be computed. The best protection of slopes appears to be dumped riprap; hand placed riprap is also discussed.

These protective covers must always be placed on a filter blanket of gravel or crushed stones, graded in such a way as not to be washed out, and allowing the wave projected water to seep down. The gradation specifications are given. These filters are necessary also in the case of cover built with concrete slabs, bituminous paving, or even of monolithic concrete covers. Design of such covers cannot, as yet, be codified. Ceramic blocks may be used; they are obtained by fusing ordinary raw bricks together. Some indications are given on planting, used chiefly against the weathering action of rain, sun, and wind.

The report underlines today's uncertainties about the actual resistance of certain sorts of covers. It is stated that full-scale experiments, in a wave tank which would produce six-foot waves, might suggest the proper design of protective covers, which would reduce the cost of earth dams now under consideration. At the present time, a wave tank which can produce waves of 1.7 ft is available, but extrapolation to six or ten-foot-high waves is not sound. Jean Goguel, France

1227. **E. N. Fox and J. McNamee, "The two-dimensional potential problem of seepage into a cofferdam,"** *Phil. Mag.*, Mar. 1948, vol. 39, pp. 165–203.

The classical methods of conformal representation for the flow of a homogeneous isotropic fluid are applied to the problem of a cofferdam embedded in an infinitely deep medium. The computations result in tables which permit the complete flow to be readily determined numerically. The theory is applicable under the assumption of known and constant permeability of an infinite medium, and a cofferdam sufficiently long to make the end effect negligible. Jean Goguel, France

1228. **R. G. Kazmann, "The induced infiltration of river water to wells,"** *Trans. Amer. geophys. Un.*, Feb. 1948, vol. 29, pp. 85–92.

The observation of the piezometric levels around an aquifer in a steady-flow condition may sometimes provide the proof that a well receives infiltrations from a surface source (lake, stream) assuring a perennial water supply. More reliable are however the pumping tests: a reversal of the slope of the piezometric surface, normally directed toward the stream, may indicate an infiltration from the stream; an examination of the slope in different directions around the well, particularly at different times during the initial pumping, may give the greatest certitude.

In the discussion, K. E. Anderson indicates a situation by which the reversal of the piezometric slope does not correspond to an infiltration from the stream. M. I. Rarabough indicates many causes of error; he shows the need of careful geological and meteorological investigations, besides the hydrometrical ones; he gives two examples, in which, owing to differences of permeability round the pumping well, the simple methods proposed by the author would fail, if not correlated with many subsidiary observations. Duilio Citrini, Italy

1229. Jean Goguel, "Distribution of stresses around a cylindrical tunnel (Répartition des contraintes autour d'un tunnel cylindrique)," *Ann. Ponts Chauss.*, Mar.-Apr. 1947, vol. 117, pp. 157–183.

Assuming the ground surface to be horizontal, the specific gravity of the rock constant, and the original stress condition hydrostatic, the author first derives the condition of stress inside the rock for an unconstrained circular tunnel section. Next, he considers the more general case when the original stress condition is not hydrostatic. Superposing upon a hydrostatic stress a homogeneous shear and a supplementary stress, the author determines the plane stress condition on an elastic basis and draws the principal stress trajectories. Stresses thus found exceed the elastic limit for deep tunnels, showing that the elasticity assumption was unsatisfactory.

The author then turns to the Huber-Mises condition of plastic deformation. If $6C^2$ is the sum of squares of the differences of principal stresses, and S the lower flow limit of the rock, then $C = S$ in the plastic zone. He assumes the deformation speeds proportional to $C-S$. Radial and tangential stresses turn out to depend linearly on $\log \rho/R$, where ρ is the distance from the axis of the tunnel, and R the radius of the cylindrical plastic zone. Since the longitduinal stress $\sigma_z = {}^1/_2(\sigma_x + \sigma_y)$ causes axial contraction, the last assumption is not yet acceptable either.

A new assumption is made, namely, that the speed of deformation is proportional to $(C - S)$ and $(\sigma_x - \sigma_o)$ ..., where σ_o is the hydrostatic stress. The total speed of deformation is the sum of the speeds of elastic and plastic deformations. The problem is determinate, but the computations are excessive and the result depends on the whole past history, in particular on the manner the tunnel was opened.

Relaxation caused by slow deformation is considered next. The basic assumptions are: (1) constant volume, (2) coincidence of the principal directions of stress and strain, (3) the principal strains are proportional to the principal stress deviations, the ratio being $\varphi(C)$; in the case $C \leq S$, $\varphi = 0$, and in the case $C > S$, $\varphi = k(C - S)/C$, respectively. The author has demonstrated in another paper that the function $\Phi = \int C\,\varphi\,(C)dC$ has to minimize the triple integral $\int\int\int \phi dV$. In the case of slow viscous deformation $\phi = C^2$, and in rapid plastic deformation $\phi = C^2 + \lambda(C - S)^2$, and C can be determined from the equation of equilibrium. After the condition of relaxation is reached, the deformation—the magnitude of which is equal to the elastic deformation—continues; experience shows that deep tunnels continue to contract.

In another chapter the author starts from Coulomb's hypothesis of failure, and demonstrates on the basis of Caquot's theorem that stresses vary with a power of the distance from the axis, and slip-surfaces have logarithmic spirals as cross sections. J. Jáky, Hungary

1230. F. H. Jackson and H. Allen, "Concrete pavements on the German autobahnen," *J. Amer. Concr. Inst.*, June 1948, vol. 19, pp. 933–976.

This article is a detailed discussion of an examination of the German superhighways. It discusses the soil conditions and climatic effects on the concrete, and gives some description of the method of construction. Comparison is made with similar highway construction in this country. As a result the authors recommend an extensive research program to improve highway construction in the USA. Frank J. Mehringer, USA

Potential or Laminar Incompressible Flow

(See also Revs. 1223, 1227, 1235, 1271)

1231. E. E. Postel and E. L. Leppert, Jr., "Theoretical pressure distributions for a thin airfoil oscillating in incompressible flow," *J. aero. Sci.*, Aug. 1948, vol. 15, pp. 486–492.

Based on the work of Theodorsen in *Nat. adv. Comm. Aero. Rep.*, no. 496, the paper presents explicit formulas for the pressure distribution along a thin wing-aileron airfoil in a uniform horizontal air stream, performing small harmonic oscillations in three degrees of freedom: vertical motion, rotation about the quarter-chord, and aileron flapping motion, the hinge axis being located at the leading edge of the aileron.

Tables and diagrams for various values of the reduced frequency are prepared, to allow rapid calculation of the pressure distribution. Heinz Parkus, Austria

1232. J. J. Moreau, "On two general theorems in the dynamics of an infinite incompressible substance (Sur deux théorèmes généraux de la dynamique de'un milieu incompressible illimité)," *C. R. Acad. Sci. Paris*, May 3, 1948, vol. 226, pp. 1420–1422.

This brief note introduces the volume vector integrals I and $-J$ of $r \times \omega$ and $r^2\omega$, where r is the position and ω the vorticity vector, and states that, in an incompressible fluid of infinite extent, $\rho dI/dt = R$, $\rho dJ/dt = H$, where R and H are the resultant vector and moment of the nongradient forces at the origin. Considerable generality and various applications to vortical flows and resistance laws are claimed for the theorems. M. J. Thompson, USA

1233. C. Bouligand, "A typical case of entrainment of a viscous fluid (Un cas typique d'entraînement d'un liquide visqueux)," *C. R. Acad. Sci. Paris*, May 19, 1948, vol. 226, pp. 1571–1573.

Equations are written for the flow of a viscous fluid within a thin torus of arbitrary cross section which begins at $t = 0$ a rotation with angular velocity $f(t)$ about its central axis of revolution. For the case of an infinite inner radius of the torus this reduces to the axial flow produced within a cylinder when the cylinder begins a given arbitrary axial movement. The solution for this case is expressed in terms of the Green's function of the cross section, and is to be calculated in terms of the appropriate orthonormal functions. The significance of the form of the result is discussed relative to the general problem of initial motions. W. R. Sears, USA

1234. **T. von Kármán and J. Valensi, "Application of the boundary layer theory to the problem of oscillations of a viscous fluid of finite weight in a U-tube (Application de la théorie de la couche limite au problème des oscillations d'un fluide visqueux et pesant dans un tube en U),"** *C. R. Acad. Sci. Paris*, July 12, 1948, vol. 227, pp. 105–106.

The authors consider the case in which the Reynolds number based on the radius of the tube and the frequency of the oscillation is large. Boundary layer theory is then applicable and, for the purpose of evaluating the loss of kinetic energy per oscillation in terms of the logarithmic decrement, the velocity can be considered uniform across the tube. Moreover, when the Reynolds number is high, the velocity distribution near the walls approximates that near an infinite flat plate oscillating in its plane in an infinite fluid. The frictional force at the wall and the work done per oscillation can thus be estimated. Equating the two expressions gives an expression for the decrement in terms of the Reynolds number which is about 12 per cent less than the observed value. G. K. Batchelor, England

Turbulence, Boundary Layer, etc.

(**See also Revs. 1207, 1253, 1279, 1291**)

1235. **L. Agostini, "Equation of the laminar boundary layer in a convergent conical channel (Équation de la couche limite laminaire dans un convergent conique),"** *C. R. Acad. Sci. Paris*, May 24, 1948, vol. 226, pp. 1684–1685.

The velocity of an incompressible fluid at the limit of the laminar boundary layer in a convergent conical channel is assumed to be given by $u_o = -kr^{-m}$, where r is the distance measured from the apex of the cone. When $R_r = (u_o|r|)/\nu$ is large, the velocity in the boundary layer $u = u_of'(z)$ can be found from the equation $f''' = ff'' + \beta(f'^2 - 1)$ in which $\beta = (2m)/(3 - m)$, $z = [3/(\beta + 2)]^{1/2} R_r^{1/2} (\theta - \theta_o)$ and $(\theta - \theta_o)$ is the angle measured from the generator of the cone. This equation is similar to that of D. R. Hartree [*Proc. Camb. phil. Soc.*, 1937, vol. 33, pp. 223–239]. F. N. Frenkiel, USA

1236. **L. Sackmann, "On the transition region of flow through pipes. Experimental study of the axial dispersion (Sur les changements de régime dans les canalisations. Étude expérimentale de la dispersion parallèle),"** *C. R. Acad. Sci. Paris*, Apr. 26, 1948, vol. 226, pp. 1343–1345.

The author presents his experimental results on transition flow in pipes as a graph of mean velocity against the linear loss of head. For each total head there are two points representing the limits of the transition region. They are computed as corresponding with the flows of Poiseuille and Blasius. The slope of the straight lines connecting the two limit points is nearly constant in the transition region and the agreement with experiment is good.

A. Craya, France

1237. **T. von Kármán, "On the statistical theory of turbulence (Sur la théorie statistique de la turbulence),"** *C. R. Acad. Sci. Paris*, June 28, 1948, vol. 226, pp. 2108–2111.

The components of turbulent velocity in isotropic turbulence of an incompressible fluid are represented by a three-dimensional spectrum of space fluctuations. A spectral function $F(k, t)$ is defined by putting $\int_0^k F dk$ equal to the kinetic energy per unit mass resulting from the harmonic components of turbulent fluctuation for which $|k_i| < k$. Here k is a variable, having the dimension L^{-1}, which measures the wave length of the fluctuations.

The energy equation for a viscous fluid is written in terms of F. By making various simplifying assumptions this theory is reconciled with theories proposed by Heisenberg, Kolmogoroff, and others. In the case of large Reynolds numbers, the use of Loitsiansky's invariant, some dimensional reasoning, and a shrewd choice of an approximation to F lead to explicit formulas for the correlation functions f and g and the one-dimensional spectral function of G. I. Taylor. These formulas are said to be in good accord with measurements of J. Lauffer in Pasadena.

Stephen H. Crandall, USA

1238. **J. A. Lewis, "Boundary layer in compressible fluid,"** *Hdqtrs. Air Mat. Comm. Dayton tech. Rep.*, no. F-TR-1179-ND, Feb. 1948, pp. 1–66.

This paper is a survey of some of the published works on boundary layers in compressible flows. A presentation of the classical development of the equations for a compressible fluid is given, followed by the specialization of these equations for the case of boundary-layer flow. Next, the monograph discusses some of the more important solutions that have been obtained for velocity and temperature distributions, skin friction coefficients, heat conduction, etc. The problems of laminar stability, turbulence, and the interaction of the boundary layer with shock waves are not considered. Francis H. Clauser, USA

Compressible Flow, Gas Dynamics

(**See also Revs. 1238, 1240, 1257, 1260, 1261, 1269, 1272, 1273, 1286, 1300, 1304**)

1239. **Marie A. Burcher, "Compressible flow tables for air,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1592, Aug. 1948, pp. 1–33.

A tabulation is presented of functions of the Mach number which are frequently used in high-speed aerodynamics. The calculated values of the functions are based on the assumption that air is a perfect gas having a specific heat ratio of 1.400. Functions are tabulated for Mach numbers from 0 to 10.0 in increments of 0.01, the interval being sufficiently small, therefore, to permit linear interpolation without introducing significant error.

John E. Goldberg, USA

1240. **S. H. Browne, L. Friedman, and I. Hodes, "A wing-body problem in a supersonic conical flow,"** *J. aero. Sci.*, Aug. 1948, vol. 15, pp. 443–452.

In this paper the authors solve the linearized potential equation for steady-state supersonic flow, using conventional conical flow concepts, for a wing-body combination composed of a slender conical body with a thin symmetrical wing, both of which extend to infinity in the direction of flow. The wings may be swept either ahead of or behind the Mach cone. The potential function is divided into symmetric and antisymmetric parts, the first of which corresponds to a flow arising from the thickness distribution with no lift, while the second arises from a superposed cross flow that determines the angle of attack and gives rise to lift. Pressure distributions are computed for both cases. In the symmetric case, the authors find that the resultant pressure is less than the sum of wing and body contributions, but never less than that of either element alone. In the antisymmetric case, the lift curve slope of the combination was found always to be less than that for either the wing or the cone alone. John R. Spreiter, USA

1241. F. Ringleb, "Approximate method of determining the pressure distribution of an adiabatic gas flow," *Hdqtrs. Air Mat. Comm. Dayton Transl.*, no. A9-T-5, 1948, pp. 1–38 (transl. from *Messerschmidt Rep.*, no. 244, Aug. 31, 1940).

This report attempts to set up a correspondence between the surface velocities on a two-dimensional body in a compressible and in an incompressible flow. The "velocity-correction" relation is based on a particular solution, given by Ringleb in an earlier paper ["Exakte Lösungen der Differentialgleichungen einer adiabatischen Gasströmung," *Z. angew. Math. Mech.*, Aug. 1940, vol. 20, p. 185], of the hydrodynamic equations of motion in the hodograph variables. A distortion of the body shape is associated with the velocity-correction formula. There are other limitations not fully discussed. The method was tested by comparing the calculated velocity distribution over an almost circular shape with the calculation of Lamla for a circular cylinder using the Rayleigh-Janzen method.

The reviewer notes that for a fuller treatment of the subject of velocity-correction formulas the reader is referred to reports by I. E. Garrick and C. Kaplan [*Nat. adv. Comm. Aero. Rep.*, nos. 789 and 790]. Neal Tetervin, USA

1242. C. Ferrari, "Supersonic flow about a sharp nosed body of revolution," *Hdqtrs. Air. Mat. Comm. Dayton Transl.*, no. A9-T-18, 1948, pp. 1–22 (transl. from *Aerotecnica*, 1936, vol. 16, no. 2, pp. 121–130).

This paper is the basic paper on the method of characteristics applied to axially symmetric supersonic potential flow. All calculations are carried out in the physical plane. This distinguishes the method from that of F. Frankl ["Supersonic flows with axial symmetry," *Izv. Artill. Akad. RKKA*, 1934, no. 1] which requires the use of the hodograph plane as well as the physical plane.

Flow around a corner and flow along a straight wall are treated. In both cases the systems of characteristic equations are derived. The characteristics of one family are approximated by parabolas, those of the other family by straight-line segments. Formulas for calculating the increments in each of the velocity components are derived. It is pointed out that the solution of the flow field around an arbitrary body of revolution can be obtained by substituting for its meridian line an inscribed broken line. The flow around the conical nose can be calculated by Busemann's method, and the flows around the corners and along the straight sides of the inscribed polygon by the methods of this paper.

The procedure is not given in sufficient detail, and no discussion of a computation problem is offered. However, further detail and a numerical illustration can be found in a later paper by the same author ["The determination of the projectile with minimum wave resistance," *R.T.P. Transl. Brit. Ministry Aircr. Production*, no. 1180 (transl. from *Atti Accad. Torino*, Nov.-Dec. 1939, vol. 75)]. Lester L. Cronvich, USA

1243. O. Laporte and R. C. F. Bartels, "An investigation of the exact solutions of the linearized equations for the flow past conical bodies," *Univ. Mich. Engng. Res. Inst. Rep.*, no. 75, Feb. 1948, pp. 1–77.

The linearized (Busemann) equations for supersonic flow past conical bodies are assumed in this paper. The authors have made use of the properties of conical flow to formulate the problem in terms of a complex variable, and have substituted exact boundary conditions for the approximate linearized boundary conditions of previous treatments reported in the literature. The equations derived are applied to determine the flow past an arrow-shaped wing with angle of attack and zero yaw, and an elliptical cone with zero angle of attack. The results are subject to the usual limitations of linearized theory. Edward N. Bowen, USA

1244. B. N. Daley and M. D. Humphreys, "Effects of compressibilty on the flow past thick airfoil sections," *Nat. adv. Comm. Aero. tech. Note*, no. 1657, July 1948, pp. 1–42.

A series of airfoils 40, 25, and 15 per cent thick were tested in the Langley 4 × 18-in. high-speed tunnel at Mach numbers up to the tunnel choking Mach number for each model. The results of the tests are presented mainly in the form of a series of schlieren photographs showing flow characteristics at supercritical Mach numbers. High-speed moving pictures of the schlieren screen were also employed to determine the variations with time of the flow picture.

On the basis of studies of the schlieren movies of the flow over the models, the flow has been divided into three general types: steady separated flow, oscillating flow, and erratic separated flow. A chart is presented relating these types of flow to the model critical Mach number and the stream Mach number. Edward N. Bowen, USA

1245. M. Schaefer, "Comparison of the stream filament theory from gas dynamics with the theory of flows with axial symmetry," *Hdqtrs. Air Mat. Comm. Dayton Transl.*, no. A9-T-8, 1948, pp. 1–5 (transl. from *T. H. Dresden Peenemunde Arch.*, no. 44/7).

Differences of two to three per cent in the case of velocities, and of twelve to thirteen per cent in that of pressures, are obtained between the values predicted by the elementary one-dimensional theory and the method of characteristics, in one special case. A. W. Wundheiler, USA

1246. W. Tollmien, "Steady two-dimensional and rotationally symmetric supersonic flows" *Hdqtrs. Air Mat. Comm. Dayton Transl.*, no. A9-T-1, 1948, pp. 1–152 (transl. from *T. H. Dresden, Peenemunde Arch.* 44/1–44/6, 1940).

This paper, consisting of six chapters, contains not only an excellent summary of known results, but also many original ideas.

In Chapter I, equations for frictionless, adiabatic, steady flows with vorticity, are derived. The special form of the equation for Crocco's stream function is given in rectangular co-ordinates for the two-dimensional case, or in cylindrical co-ordinates for the rotationally symmetrical case. The theory of characteristics is treated in Chapter II for the general case of a quasilinear partial differential equation of the second order in two variables. Three ways of introducing the concept of characteristics are given, and the characteristic differential equations are discussed. Two graphical-numerical methods for obtaining approximate solutions of the characteristic equations are presented—the method of lattice points and the method of meshes.

Chapter III treats the two-dimensional potential flows by the general method of characteristics. The relation between the characteristics in the physical plane and in the hodograph plane is shown. The method is applied to the flow around a biwing and to the calculation of the pressure change around a small turn. The lift and drag coefficients for a circular arc profile are also determined.

In Chapter IV, the method of characteristics is extended to axially symmetric problems. The method of meshes is used. Tables are given to facilitate computations and examples of flows through a diffuser and through a nozzle are worked out in detail. Stationary shocks are treated in Chapter V. A parametric representation of the shock relations is formulated; it yields a shorter presentation of some theoretical results and lends itself readily to calculations.

Finally, in Chapter VI, the characteristic equations for the cases of plane and axially symmetric flow with rotation are derived. C. T. Wang, USA

1247. **S. V. Falkovich, "Plane motion of gas at hypersonic velocity"** (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, July-Aug. 1947, vol. 11, pp. 459–464.

For plane, steady, irrotational, adiabatic flows the equation for the Legendre potential $\phi = ux + vy - \phi$ (ϕ — velocity potential) is shown to be of the Darboux type

$$\phi_{\lambda\mu} - L(\mu - \lambda)(\phi_\lambda - \phi_\mu) = 0 \quad [1]$$

where λ, μ are the Mach variables. L is a function equal to $2/(\mu - \lambda)$ for small $\mu - \lambda$, that is, for large Mach numbers (say, greater than 4). In this case Equation [1] can be solved by the formula

$$\phi_{\lambda\mu} = \frac{\partial^2}{\partial\lambda\partial\mu}\frac{X(\lambda) - Y(\mu)}{\lambda - \mu}$$

where X and Y are two arbitrary functions.

The general expressions for x, y, in terms of λ, μ, ϕ_λ, ϕ_μ, and the speed w, are given and computed in terms of X, Y, their derivatives, λ and μ. It is shown that the equation for the stream function, at large Mach numbers, is also of type [1], with $M = 3/(\mu - \lambda)$.

In a last section a very simple derivation of the similarity index for transonic flows, $(\delta/l)(M^2-1)^{3/2}$, and Tsien's similarity index for hypersonic flows, $(\delta/l)(M^2-1)^{1/2} \approx (\delta/l)M$ are given.

A. W. Wundheiler, USA

(Editor's Note: A ten-page translation of this paper has been made by N. Precoda; the editor can supply photostats at the usual rate.)

1248. **A. H. Taub, "Relativistic Rankine-Hugoniot equations,"** *Phys. Rev.*, Aug. 1, 1948, vol. 74, pp. 328–334.

The speed of sound can exceed that of light for an appropriate choice of the internal energy function. From a kinetic theory, the paper derives the relativistic flow and shock equations, as well as an expression and an inequality for the internal energy ϵ. This inequality implies that $\delta = c_p/c_v < 2$, which insures that both the sound and shock velocity are smaller than that of light; they would reach this limit for $\delta = 2$ and $\epsilon = (1/\delta - 1)p/\rho$.

A. W. Wundheiler, USA

1249. **D. G. Samaras, "Gas dynamic treatment of exothermic and endothermic discontinuities,"** *Canad. J. Res. Sec. A*, Jan. 1948, vol. 26, pp. 1–21.

The flame front is defined as a thin zone where the combustion takes place or, in the case of premixed gas, where the burned and unburned gases come into contact. The known physical aspects of the combustion are summarized.

The flame front, as an oblique shock wave, is investigated theoretically; the influence of viscosity, heat conduction, and radiation are neglected. The author starts from the well-known equations of the shock wave. In the energy equation a new element is introduced, "a heat addition function", to take into account the heat content of the combustible mixture. The greatest part of the paper is devoted to relations among the densities, pressures, and velocities of the cold and hot parts of the flame front. It seems to be an open question how far the results can be confirmed by experiments.

The most important conclusions are: An endothermic discontinuity behaves always as a compression shock. An exothermic discontinuity behaves as a shock wave, when the entry normal Mach number is greater, and as an expansion when the entry normal Mach number is less than unity.

E. Abody-Anderlik, Hungary

1250. **M. Schaefer, "Propagation of a two-dimensional irrotational supersonic flow along a wall,"** *Hdqtrs. Air Mat. Comm. Dayton Transl.*, no. A9-T-10, 1948, pp. 1–13 (transl. from *T. H. Dresden Peenemunde Arch.*, no. 44/9, Nov. 5, 1942).

Relations are derived connecting the conditions along a Mach wave with those along a streamline in a two-dimensional supersonic potential flow.

Hideo Yoshihara, USA

1251. **H. Mirels, "Theoretical wave drag and lift of thin supersonic ring airfoils,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1678, Aug. 1948, pp. 1–20.

This is an application of the linearized theory to ring airfoils. If the ratio of chord to diameter is small, the surface lying in the forward Mach cone of any point of the wing over which the integration of source or doublet strength is carried, is nearly plane. Then the simple proportionality of singularity strength and normal velocity used for flat (or nearly flat) wings is still approximately correct. Thus the calculation is no more complicated than for any twisted rectangular airfoil. Naturally, cylindrical co-ordinates are used.

The author considers lift and wave drag of ring airfoils of arbitrary profile. In particular, he treats those of symmetrical diamond profile and cylindrical basic shape, and calculates maximum lift/drag ratios, including skin friction. To assess the accuracy of the method, a comparison is made between these results (specifically, the lift on the outer surface) and those of Brown and Parker [*Nat. adv. Comm. Aero. wartime Rep.*, no. L-720, 1946], who calculated the linearized flow outside (but not inside) open-nose bodies of revolution without the approximation involved here.

W. R. Sears, USA

1252. **R. Sauer, "The method of characteristics for the one-dimensional unsteady flow of a gas,"** *Hdqtrs. Air Mat. Comm. Dayton Transl.*, no. A9-T-2, 1948, pp. 1–26 (transl. from *Ingen.-Arch.*, 1942, no. 13, p. 79).

Following the earlier theoretical work of Riemann [Gesammelte Werke, 2nd ed., p. 156] on the one-dimensional unsteady flow of a nonviscous gas, and more recent writers, the author treats the practical problem of determining a one-dimensional unsteady flow of a gas for given initial or boundary conditions by means of a graphical method of approximation. The method is based on the theory of characteristics, as is the Prandtl-Busemann method for a two-dimensional steady supersonic flow. The method holds for subsonic as well as for supersonic flows. Applications are made to unrestrained flows produced by the motion of a piston in a semi-infinite tube, a finite tube with a piston closed on one end, a finite tube with two pistons, and the propagation of a disturbance in an infinite tube. Only continuous adiabatic changes of state are considered.

Abe Gelbart, USA

1253. **B. Chaix, "The efficiency of compression by means of stationary shock waves in supersonic gas flow (Le rendement de la compression par ondes de choc stationnaires dans un gaz animé d'une vitesse supersonique),"** *Schweiz. Arch.*, 1948, vol. 14, Feb., pp. 39–49; June, pp. 161–169.

The performance of supersonic diffusers is separated into two sections: (*a*) The efficiency of the shock system, neglecting the influence of the boundary layer, and (*b*) the effect of boundary-layer interaction with the shock waves in the main flow. The shock-wave systems under consideration include the normal shock, the normal shock followed by a subsonic diffuser, the reversed supersonic nozzle followed by a subsonic diffuser, and the multishock diffuser of K. Oswatitsch.

Under heading (*a*) the theoretical analysis is based on simple shock theory, but empirical constants are introduced to allow for the unknown efficiency of the subsonic diffuser.

In the second case a quantitative analysis is made of the increase of boundary-layer thickness in passing normal and oblique shocks. The main assumptions made in this calculation are: (1) The pressure and total energy distributions at any cross section are uniform throughout the main flow and boundary layer, and (2) the distribution of velocity is known in the boundary layer upstream and downstream of the shock. There appears to be good correspondance between the theory and available test results. However, no general comparison is given of the over-all theoretical efficiencies with the experimental ones.

G. M. Lilley, England

Aerodynamics of Flight; Wind Resistance

(See also Revs. 1180, 1194, 1215, 1231, 1240, 1244, 1251, 1270)

1254. H. Roberts, "Rolling performance of aircraft," *Aircr. Engng.*, June 1948, vol. 20, pp. 167–170, 181.

Theoretical formulas and curves are given for the rolling moment derivatives, l_p and l_ξ, for elliptic, straight tapered, and trapezoidal wing planforms. The method is based on a modified form of the Schrenck formula for distribution of lift over such surfaces. Comparison with experimental curves for the derivatives shows that the method gives good agreement, and enables reasonably accurate estimates of the rolling performance of an airplane to be made. A simple method is also given for transforming a wing, the planform of which has a straight center section and tapered outer section, into a straight tapered wing, to facilitate rapid computation of the above mentioned derivatives. It is also shown that, for this purpose, an elliptical wing may be assumed to be of straight tapered planform with a taper ratio of $^1/_3$.

E. Arthur Bonney, USA

1255. L. Sternfield and M. O. McKinney, Jr., "Dynamic lateral stability as influenced by mass distribution," *J. aero. Sci.*, July 1948, vol. 15, pp. 411–417.

The paper presents a summary of some recent theoretical and experimental investigations of the effects of mass distribution (such as rolling or yawing moments), wing incidence, wing loading, and altitude on the stability of the lateral oscillations of airplanes. The results are presented in the form of charts that show calculated oscillatory stability boundaries and time histories of the lateral oscillations of models in flight in the Langley free-flight tunnel.

On the basis of these lateral stability studies, the preparation of general stability charts for the prediction of the stability of the lateral oscillations does not appear feasible. A small variation in some of the mass and aerodynamic parameters may cause such a pronounced change in the oscillatory stability that a special stability analysis should be made for each airplane.

Andrew Vázsonyi, USA

1256. K. H. Nordenfelt, "Ballistic experiments and air resistance (Ballistiska erfarenheter av luftmotstandet)," *Tekn. Tidskr.*, Feb. 22, 1947, vol. 77, pp. 185–188.

From observed values of projectile drag the drag coefficient K_D of an airplane is calculated and found to afford a good junction between the values of K_D in the subsonic region on one side and the supersonic region on the other side as obtained from aerodynamic theories. A discussion of test results is presented.

E. F. Lype, USA

1257. W. A. Tucker and R. L. Nelson, "Characteristics of thin triangular wings with constant-chord partial-span control surfaces at supersonic speeds," "*Nat. adv. Comm. Aero. tech. Note*, no. 1660, July 1948, pp. 1–43.

Employing methods based on the linearized theory of supersonic flow, an analysis is made of the characteristics of constant-chord partial-span control surfaces on thin triangular planform wings at supersonic speeds. Two cases are treated: In one the flap extends outward from the center of the wing, and in the other the flap extends inward from the wing tip. The wings considered have their leading edges both ahead of and behind the Mach cone. Expressions are found for the lift coefficient, rolling moment coefficient, and hinge-moment coefficient due to flap deflection, the hinge-moment coefficient due to angle of attack, and the pitching-moment coefficient due to flap lift. Illustrative examples are appended.

H. Julian Allen, USA

1258. J. G. Lowry and L. E. Schneiter, "Investigation at low speed of the longitudinal stability characteristics of a 60-deg sweptback tapered low-drag wing," *Nat. adv. Comm. Aero. tech. Note*, no. 1284, May 1947, pp. 1–55.

Various wind-tunnel tests were made in order to obtain more information about swept wings with an angle of sweep greater than 45 deg, and to improve the longitudinal stability of highly swept wings. The plain 60-deg swept wing was tested and compared with a wing with deflectable tips, various flap conditions, numerous changes in planform, various leading edge slats, and reduced aspect ratio.

For the plain wing, an increase in stability occurred, amounting to a rearward shift in neutral point of about 14.4 per cent of the mean aerodynamic chord, at a lift coefficient of 0.2. At a lift coefficient of 0.5 the stability decreased, and the wing became extremely unstable.

Two of the conclusions reached were: (1) The longitudinal stability of the swept wing can be improved by the addition of an extension at the leading edge, and (2) swept wings of aspect ratio near one had better longitudinal stability characteristics than wings with higher aspect ratios.

Tuft studies were also made and the report uses these pictures to assist in the analysis of the results.

H. R. Gillespie, Jr., USA

1259. J. M. Evans and P. T. Fink, "Stability derivatives—determination of l_p by free oscillations," *Austral. Counc. Aero. Rep.*, no. 34, Apr. 1947, pp. 1–11.

This paper describes a fundamental approach to an increasingly important problem, that of experimentally measuring the parameters which determine the full-scale dynamic behavior of an aircraft. Wind-tunnel measurements were made by allowing one degree of freedom, rotation in roll, with known stiffness and inertia, and recording the decay of free oscillations. The mechanical apparatus, recording equipment, and test procedure are fully described.

The tests were confined to one derivative, the damping in roll, l_p. The technique, however, could logically be extended to measure other parameters which affect the dynamics of aircraft, including damping derivatives, stiffness derivatives, and moments of inertia. Also, by varying the frequency of the oscillations, the tests yielded experimental data which would be useful in substantiating the oscillating airfoil theory. This would be a definite advantage of the free oscillation method over the forced oscillation or rotary arm technique. (It would seem worth while to derive and perfect wind-tunnel techniques for measuring all important derivatives, and substantiate the measurements by full-scale flight measurements.)

G. F. Campbell, USA

1260. **V. I. Stevens, Jr., "Prediction of load distribution and its effect on aerodynamic characteristics at subsonic speeds for wings of arbitrary planform,"** *Aero. Engng. Rev.*, June 1948, vol. 7, pp. 37–42.

This paper discusses the subsonic aerodynamic characteristics of wings of widely diversified planforms. The general range of variables includes sweep of from 45 deg forward to 60 deg back, aspect ratio of 1.5 to 8.0, and taper ratios of 0 to 1.5. The theoretical and experimental values of the lift curve slope and aerodynamic center location for a number of random planforms are correlated in two separate figures. The theoretical results are quoted from a paper by De Young [*Nat. adv. Comm. Aero. tech. Note*, no. 1491, Dec. 1947] while the measured results are determined from wind-tunnel measurements. In all cases the agreement is fairly good. Wei-Zang Chien, China

1261. **J. C. Evvard, "Theoretical distribution of lift on thin wings at supersonic speeds (an extension),"** *Nat. adv. Comm. Aero. tech. Note*, no. 1585, May 1948, pp. 1–49.

This is an extension of the author's previous method [*Nat. adv. Comm. Aero. tech. Note*, no. 1382, July, 1947] of the linearized supersonic flow (point-source distribution and fictitious diaphragm extension ahead of the subsonic leading edges). In this paper a unique solution is obtained for the subsonic trailing edge by assuming that in the supersonic flow field the Kutta-Joukowski condition is still satisfied at the subsonic trailing edge.

The method is applied to evaluate the lift distribution of a thin flat-plate wing having a straight leading edge and an arbitrarily curved wing tip with subsonic edges. The supersonic velocity potential is obtained for the regions influenced by the subsonic trailing edge. The solutions are found which satisfy the unique Kutta-Joukowski condition of no velocity discontinuities at the subsonic trailing edge, or allow a given discontinuity to exist in the cross-velocity component in the wake of the subsonic trailing edge (corresponding to shed vortex sheets producing sidewash in the plane of the wing). The method is also applied to compute the upwash and the perturbation velocities in the plane of the wing at the subsonic wingtip of a sweptback trapezoidal wing.

E. V. Laitone, USA

1262. **J. M. Evans, "Stability derivatives—wind-tunnel interference on lateral derivatives l_p, l_r, and l_v with particular reference to l_p,"** *Austral. Counc. Aero. Rep.*, no. 33, Mar. 1947, pp. 1–18.

The interference rolling moment on a wing rolling in a wind tunnel was computed for three straight tapered wings at three aspect ratios for circular cross sections, and at an aspect ratio of 6 for a rectangular cross section.

For a circular tunnel the method of images was used after replacing the wing by a bound vortex of length equal to the span, and a system of semi-infinite trailing vortexes. The induced rolling moment, caused by the image trailing vortex, was determined and the correction factor K obtained. It was shown that K for the circular tunnel with open section is equal in magnitude but opposite in sign to that for the closed tunnel. For a rectangular tunnel the interior of the rectangle was transformed by conformal mapping into the interior of a circle with an "equivalent" wing. Although the planform and lift distribution of the original wing were distorted by this process, the effect was found to be negligible so that the results for the circular tunnel could be applied.

It was found that tunnel interference increases the apparent l_p for closed tunnels and reduces it for open tunnels.

I. Statler, USA

1263. **C. H. McLellan and J. I. Cangelosi, "Effects of nacelle position on wing-nacelle interference,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1593, June 1948, pp. 1–87.

The authors report on experimental work in which an optimum position with regard to critical Mach speed for lift and drag was sought for a nacelle at a given spanwise station on a wing. The variables considered were the vertical position relative to the wing, fore and aft position, and angle of incidence. The tests were conducted at Mach speeds up to 0.7, and with only minor modifications of the wing and nacelle components.

The low position at zero incidence and with the 50 per cent wing chord point somewhat forward of the 50 per cent nacelle axis point (nacelle nose 66 per cent of wing chord forward of the wing nose) appeared best. The logical and reasonable conclusion that good high-speed aerodynamic design components should be used is not supported by experimental evidence either presented or cited.

The pressure data presented should be useful in checking the methods that are employed for estimating critical Mach speeds in the case of interference problems. The data contain additional evidence that local critical Mach speeds may remain localized and unimportant.

The influence of nacelle position and attitude on critical Mach speed with regard to pitching moment was apparently not measured and hence this work must be regarded as incomplete for design purposes. M. G. Scherberg, USA

1264. **Felicien F. Fullmer, Jr., "Wind-tunnel investigation of a systematic series of modifications to a flying-boat hull,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1576, May 1948, pp. 1–35.

Wind-tunnel tests, conducted on a representative flying-boat hull of length-beam ratio equal to 6.7, indicate possible aerodynamic drag reductions through reduction of step depth, and variations in the shape and location of the forebody and afterbody chines. The investigation was made at a Reynolds number of 6.4 $\times 10^6$ based on hull length. The hull was mounted on a wing which completely spanned the tunnel.

The hull drag coefficient (based on hull frontal area) was 0.090 for the basic configuration, for all step depths from 8 to 16 per cent of the beam. Reducing the step depth to 4 per cent of the beam lowered the hull drag 20 per cent. Complete elimination of the step produced no further reduction in drag. Rounding a portion of the forebody or afterbody chine, either separately or together, produced 18 per cent decrease in drag. Elimination of all sharp chines, step and discontinuities reduced the drag coefficient 30 per cent, of which two thirds is attributed to the step.

Ernest G. Stout, USA

1265. **S. Wynia, "Calculation and correction of the take-off distance of a propeller airplane. Part 1: Formula for the take-off distance and derivation of an approximation for use as correction (Berekening en correctie van de aanlooplengte van schroefvliegtuigen. Deel 1: De formule voor de aanlooplengte en de afleiding van een benaderingsformule voor correctiedoeleinden),"** *Nat. LuchtLab. Amsterdam Rap.*, no. V.1300, May 1947, pp. 1–15.

It is proposed in this paper to determine the take-off distance of a propeller-driven airplane from the observed performance of a similar airplane, correcting it for different conditions of wind velocity, weight, engine power, air temperature, barometric pressure, humidity, runway friction, and inclination of runway. For this purpose the take-off distance is obtained by integration of the equations of motion and is employed to derive the correction factors for the different conditions mentioned.

F. Hymans, USA

1266. B. Göthert, "Comparison of drop and wind-tunnel experiments on bomb drag at high subsonic speeds," *Nat. adv. Comm. Aero. tech. Memo.*, no. 1186, May 1948, pp. 1–12 (transl. from *Dtsch. Luftfahrtforsch. Forschungsber.*, no. 1570, Apr. 17, 1942).

Full-size bombs were dropped from 11 km, and the variation of drag coefficient with Mach number was obtained from the bomb trajectory. The highest Mach number reached was 0.97. As the speed of sound was approached, the drag coefficients of models of the full-size bombs, obtained from tests in a closed-throat wind tunnel up to a Mach number slightly greater than 0.90, increased at about the same rapid rate as the drag coefficients obtained from the drop tests. The drag coefficients from the tunnel tests, however, were greater than those from the drop tests.

As the speed of sound was approached, the drag coefficients from tests of models of the bombs at low Reynolds numbers in an open-throat wind tunnel increased at a smaller rate than the drag coefficients from the drop tests. The author suggests the small Reynolds numbers and lack of tunnel wall corrections for the open-throat tunnel data as possible causes of the large differences between the open-throat tunnel data and the drop test data.

Neal Tetervin, USA

1267. L. E. Schneiter and J. M. Watson, "Low-speed wind-tunnel investigation of various plain-spoiler configurations for lateral control on a 42-deg sweptback wing," *Nat. adv. Comm. Aero. tech. Note*, no. 1646, June 1948, pp. 1–29.

Sweptback wings on high-speed airplanes have aggravated the problem of securing adequate and efficient lateral control. Spoilers (which may be defined as segments thrust into the air stream from the upper surface of a wing) used as lateral-control devices have the advantage that favorable yawing moments are exerted as compared with ailerons, and that full-span high-lift flaps can be used along with them.

This report covers some exploratory tests at 0.18 Mach number on a 64_1-112 airfoil with 40 deg sweepback of the leading edge. The best spoiler configuration consisted of several segments located near the trailing edge of the wing, each segment being perpendicular to the air stream. Some of the objectionable characteristics of control by spoilers alone are mentioned, but these can probably be neutralized by using the spoilers in combination with ailerons. Further tests at higher speeds and with variations in wing construction need to be made

W. C. Johnson, Jr., USA

1268. C. Ferrari, "Interference between wing and body at supersonic speeds—theory and numerical application," *J. aero. Sci.*, June 1948, vol. 15, pp. 317–336.

The aerodynamic interference between a body of revolution with pointed nose and a rectangular wing carried by it under supersonic conditions is dealt with from the point of view of linearized theory. The fundamental concept of the analysis is that owing to the presence of the wing, a field of flow is set up which neutralizes the normal velocities induced by the body at the wing, and conversely, that owing to the presence of the body a field of flow is set up which balances the normal velocities induced by the wing at the body. Secondary effects—of the wing on itself, owing to the presence of the body, and of the body on itself, owing to the presence of the wing—are neglected in the analysis.

The normal velocity induced by the wing at the body is resolved into harmonic components in terms of cylindrical coordinates, and the effects produced by the separate components are calculated by an application of the method of von Kármán and Moore. The normal velocity induced by the body at the wing is similarly (and somewhat less naturally) resolved into "harmonic components" in terms of the longitudinal and lateral co-ordinates of the wing. The limiting case of an infinitesimally narrow chord wing is also considered.

A numerical application is calculated. It is found that the interference lift produced by the body on the wing considerably exceeds that produced by the wing on the body, while the opposite is true with regard to the pitching moments. However, the method requires a separate calculation for each particular case, and it would appear that the computation of additional examples is necessary to permit any perfectly general conclusions to be drawn from this analysis. An optimum angle of wing setting for least drag is also determined. It may serve as a rough guide in practical cases.

The reviewer notes the related work of S. Kirkby and A. Robinson, *Coll. Aero. Cranfield Rep.*, no. 7, 1947, and S. H. Browne, L. Friedman, and I. Hodes, *J. aero. Sci.*, 1948, vol. 15, no. 8.

A. Robinson, England

Aeroelasticity (Flutter, Divergence, etc.)

(See also Rev. 1231)

1269. B. Smilg, "An engineering evaluation of flutter and other aeroelastic problems at transonic and supersonic speeds," *ATI tech. Data Dig.*, July 15, 1948, vol. 13, pp. 9–18.

This paper is a comprehensive review and presents what appear to be the critical aeroelastic problems of transonic flight. The author indicates that the conclusions of the paper are of a preliminary nature, and that final conclusions must await further experimental and theoretical investigations.

The results are summarized as follows: (1) The prevention of classical flutter is likely to be less difficult at supersonic speeds than at either subsonic or transonic speeds. (2) The prevention of aileron reversal may be the determining factor for torsional rigidity requirements of supersonic wings. (3) Divergence is not likely to be critical for airfoils to which trailing-edge flap types of control surfaces are attached. (4) The reduction in torsional rigidity requirements for high altitude flight may permit a missile to fly at supersonic speed at high altitude with no greater torsional rigidity than is needed for launching. (5) The dynamic loads induced by a given gust are likely to be more critical in the lower transonic speed range than at subsonic or supersonic speeds. Increasing the wing loading and the altitude decreases the gust loads.

The aeroelastic problems encountered may be minimized by utilizing the following design features: High torsional rigidity of airfoils and control surfaces; irreversible controls or dampers in the controls; and placing the center of gravity and elastic axis of the airfoil as far forward as possible.

Arthur A. Regier, USA

1270. R. Rosenbaum and R. H. Scanlan, "A note on flight flutter testing," *J. aero. Sci.*, June 1948, vol. 15, pp. 366–370.

The rate of decay of an unsymmetrical oscillation, generated by an abrupt aileron deflection-and-release, has been drawn from oscillograph flight records. The agreement with theory is reasonable. The lag function of aerodynamic forces is assumed to hold in the case of damped oscillations.

J. H. Greidanus, Holland

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 1176, 1187, 1215, 1249, 1281, 1287, 1289, 1292, 1301, 1302)

1271. **C. Ferrari, "On the determination of the 'best' profile of blades for axial compressors (Sulla determinazione del profilo 'ottimo' per le pale dei compressori assiali),"** *R. C. Accad. Lincei*, May 1947, vol. 2, pp. 576–586.

The "best" profiles for axial compressors are those having an even pressure distribution on both sides for given lift coefficient, blade cascade angle, and pitch-to-chord ratio. In the present paper the best profile is calculated under the assumption that the velocities tangential to the blade contours are constant on both sides of the blade profiles. This corresponds to a uniform pressure distribution. It is further assumed that the thickness and the camber of the profiles are small, and the flow frictionless. The application of the equations is shown by means of two examples. The method developed should give good results for compressors with small stage pressure rises. However, with high stage pressure rises, and correspondingly cambered blade profiles with high thickness ratios, the method probably gives a rough approximation only, especially as the effects of friction are considerable under these conditions.

E. Haenni, USA

1272. **A. R. Howell, "The aerodynamics of the gas turbine,"** *J. roy. aero. Soc.*, June 1948, vol. 52, pp. 329–356.

This paper, emphasizing the aerodynamic side of the matter, deals with the design of compressors and turbines as they are used for aircraft engines, and concentrates on internal flow through axial gas turbines. Data on lift and drag coefficients tested in cascades and stages are given. Knowledge of such coefficients is used to determine the efficiency and bulk of compressor and turbine components in gas turbines.

Three-dimensional flow in axial compressors and turbines is briefly mentioned to show how effects of boundary layer and secondary flow have to be considered when results of cascade tests are applied to single and multistage designs. Effects of high operating Mach number are mentioned and illustrated by a compressor cascade characteristic. Centrifugal compressors are dealt with only sketchily, as they are not covered by the author's theoretical and experimental methods for blading design. Some general design considerations are finally given, and different combinations of compressors of both types with turbines are discussed. The engine aerodynamicists interested in details will find a large appendix of references in this report.

E. Mühlemann, Switzerland

1273. **H. Woodhouse, "Inlet conditions of centrifugal compressors for aircraft engine superchargers and gas turbines,"** *J. aero. Sci.*, July 1948, vol. 15, pp. 403–406.

The entrance angles of a centrifugal impeller are frequently based on an inlet velocity diagram, but it has long been known that the theoretical angle so determined must be increased for best efficiency. The amount of this increase (called by the author "angle of attack") is not critical for low impeller tip velocities, but has a clearly defined optimum at tip velocities greater than about 1400 fps for air, and at comparable Mach numbers for other fluids. Certain experimental data indicate that it should be about four degrees. In constructing the velocity diagram, due allowance must be made for the change of density with high inlet velocities, as well as for the thickness of the blades. A chart is given to facilitate calculation of the absolute inlet velocity.

C. W. Smith, USA

1274. **A. W. Morley, "Equilibrium running of the simple jet-turbine engine,"** *J. roy. aero. Soc.*, May 1948, vol. 52, pp. 305–322.

In this paper, written prior to March 1947, the author presents an outline of the steps involved in determining the possible operating points of a simple turbojet by matching its various components. The analysis is carried out in terms of nondimensional parameters which, by now, have become well known. The link between engine and aircraft parameters is also discussed briefly.

Andrew Fejer, USA

1275. **A. Kahane, "Investigation of axial-flow fan and compressor rotors designed for three-dimensional flow,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1652, July 1948, pp. 1–58.

In the conventional method of design of axial-flow fan blading, the blades are loaded so that no mutual interference exists between adjacent blade elements. This loading is accomplished by having the absolute tangential velocities vary inversely as the radius. Rotors designed in such manner are known as "free vortex rotors."

A higher pressure rise may be obtained by loading the rotor at the tips higher than the corresponding free vortex loaded rotor, but the flow becomes three-dimensional. An approximate method is given whereby available two-dimensional cascade airfoil data may be applied to this three-dimensional flow.

Two three-dimensional-flow rotors were designed. One with approximately uniform downstream tangential velocity distribution, and one with approximately solid-body downstream velocity distribution. These rotors were tested in the Langley propeller-research tunnel.

The results indicate that: (1) High-efficiency axial-fan and compressor blading may be designed by incorporating three-dimensional flows; (2) the three-dimensional theory used in conjunction with two-dimensional cascade data is sufficiently accurate for design purposes; (3) the tip-clearance losses of rotors loaded highly at the tips are not excessive.

Karl E. Schoenherr, USA

Experimental Flow Equipment and Technique

(See also Revs. 1236, 1259, 1262, 1266, 1270, 1282, 1285, 1292, 1298)

1276. **R. Betchov, "Influence of the thermal conduction upon the hot-wire anemometers (L'influence de la conduction thermique sur les anémomètres à fils chauds),"** *Proc. kon. Ned. Akad. Wet.*, June 1948, vol. 51, pp. 721–730.

This paper is the third of a series by the same author on the same subject.

The sensitivity of a hot-wire anemometer is reduced by heat conduction towards the supports, since both amplitude and phase are modified in the case of fluctuations. Calibration of the wire by means of a pulsating electric current may lead to the same errors. Yet it is possible to calculate the errors after measuring certain coefficients.

The author establishes first the general differential equation of a hot wire with heat conduction towards the supports. Then he discusses the stationary solution, the effect of fluctuations of the airflow, of the electric current, and of the ambient temperature. Finally he gives an approximate calculation of the correction. Some plots are added as examples for the calculation procedure.

Wilhelm Spannhake, USA

1277. R. M. Mains, "A strain-gage balance system for a supersonic wind tunnel," *Proc. Soc. exp. Stress Anal.*, 1948, vol. 5, no. 2, pp. 100–114.

The development of a strain-gage balance for the Lone Star Laboratory tunnel is described. Lift, drag, side force, pitching and yawing moments are obtained from the strains in two annular diaphragms carrying the model's tail strut inside a fairing. The work is noteworthy in that the usual failings of such balances—drift with time and temperature change, interaction of lift with drag—have been recognized and minimized.

R. Smelt, England

Hydraulics; Transport of Solids; Cavitation

(See also Revs. 1223, 1224, 1228, 1234, 1236, 1287, 1292, 1302)

1278. Ed Brun, L. Demon, and M. Vasseur, "Mechanical capturing of particles in suspension in the air (Captation mécanique de corpuscules en suspension dans l'air)," *Rech. aéro. Paris*, Jan. 1948, no. 1, pp. 15–19.

The methods of measuring drop size distribution in fog and cloud have been fraught with difficulty and tedium. One of the most accurate ways, that of electrical precipitation, is too delicate for everyday use. But it has been employed by the authors of the present paper as a check on a refined mechanical technique.

The simplest mechanical method, that of placing an oil-coated plate in an airstream, selects highly in favor of the large-size droplets. The authors calculate theoretically the capture coefficient (ratio of given-sized drops captured to those passed) as a function of airstream speed, plate width, and drop size. Other things being equal, it is shown that the capture coefficient is the higher the narrower the plate, and approaches unity as the width approaches drop dimensions. This fact suggests the use of a network of very fine fibers. Plexiglas fibers were tried and the results found to compare favorably with the electrical methods. Much caution, however, is still required in employing such direct-counting techniques, and indirect means, such as those utilized by the recently considered optical instruments, appear to be more promising in the long run.

Joanne Starr Malkus, USA

1279. L. Sackmann, "On transitions in pipe flows. Theory of the dispersion of characteristics (Sur les changements de régime dans les canalisations. Théorie de la dispersion des caractéristiques)," *C. R. Acad. Sci. Paris*, Apr. 19, 1948, vol. 226, pp. 1248–1250.

This communication is the third of a series in which the author investigates the transition from laminar to turbulent flow in pipes. He has previously shown that these transitions are periodic, and measured the maximum and the average discharges. Now he introduces a function of the total loss of head, descriptive of the laminar and turbulent states possible with the same loss of head.

A. Craya, France

1280. Kalervo Aimonen, "Graphical determination of flow rate of water (Grafisk beräkning av strömningsförhållanden hos vatten)," *Tekn. Tidskr.*, Jan. 25, 1947, vol. 77, pp. 91–95.

The author presents diagrams for nonuniform flow conditions of water in rivers, etc., based on the Tolkmitt and Ganguillet-Kutter formulas. The accuracy of the results is discussed.

The reviewer notes that the Ganguillet-Kutter formula, hardly considered correct any more, is gradually being replaced by better and less complicated ones.

Hans Thygesen Kristensen, Denmark

1281. L. G. Bell, "Some model experiments on the effect of blade area on propeller cavitation," *Shipbuilder*, Apr. 1948, vol. 55, no. 472, pp. 287–292.

A family of six screws of differing blade areas and constant face pitch, blade shape, and thickness ratio have been tested in the Haslar cavitation tunnel. For each experiment the tunnel water speed was adjusted to a fixed value, and the required cavitation number was obtained by varying the coaming pressure. Slight or moderate cavitation results in a progressive decline of thrust and efficiency.

Increase of blade area tends to reduce the amount of cavitation, but it also reduces the efficiency at lower (noncavitating) speeds. Thus, for any given cavitation number, we should expect to find an optimum area for best efficiency. This feature is shown in a diagram of efficiency versus blade area ratio. The author presents also a study of the spread of laminar cavitation under decreasing pressure.

L. J. Tison, Belgium

1282. R. Cerf, "Motion and deformation of an elastic spherical particle in viscous flow with constant velocity gradient (Mouvement et déformation d'une particule élastique sphérique dans un écoulement visqueux à gradient de vitesse constant)," *C. R. Acad. Sci. Paris*, May 19, 1947, vol. 226, pp. 1586–1588.

The present paper is concerned with the orientation and deformation of elastic ellipsoidal particles in suspension in a shear flow of constant velocity gradient. Several theorems, derived on the basis of Stokes's approximation to the hydrodynamic equations, are stated without demonstration. These results are applicable to a quantitative study of the phenomena of birefringence in the motion of suspensions of deformable particles.

Louis Landweber, USA

1283. Charles Jaeger, "Total impulse and its relation to the total energy of liquid flow with a free surface (De l'impulsion totale et de ses rapports avec l'énergie totale d'un courant liquide à surface libre)," *Rev. gén. Hyd.*, 1947, vol. 13, Jan.-Feb., pp. 12–19; Mar.-Apr., pp. 86–87; May-June, pp. 143–151; July-Aug., pp. 191–197; Sept.-Oct., pp. 257–261.

In a penetrating and thought-provoking analysis of open-channel flow, the author investigates in considerable detail the interdependence of the four major flow variables: the rate of discharge, the depth, the total energy (total head referred to the channel bed), and the total impulse (momentum flux plus pressure). A general equation is developed which relates these variables in terms of energy and momentum coefficients for both velocity variation and flow curvature. Numerous types of flow phenomena are then studied in the light of particular forms of the general equation, and the results are used to clarify many an apparent paradox.

The analysis is admittedly so far beyond the usual simplified concepts of elementary hydraulics that few experimental data of sufficiently complete a nature are yet available for quantitative verification of the author's conclusions. Moreover, certain of his deductions are too novel to be accepted in their entirety without careful study and test. As a whole, nevertheless, the paper warrants the attention of all who are engaged in open-channel research, whether for its extensive annotation and history review, its illuminating comments upon many a feature of free-surface flow not often considered, or its by no means unsuccessful attempt to include all such phenomena in a single unified treatment. The author has written a similar article in English ["Steady flow in open channels: the problem of Boussinesq," *J. Instn. civ. Engrs.*, Feb. 1948, vol. 29, p. 338 (see Rev. 1054, Applied Mechanics Reviews, June 1948)].

Hunter Rouse, USA

1284. **E. Hoeck, "Loss of pressure in the water conduit of big hydroelectric power plants (Pertes de charge dans les conduits forcées des grandes centrales hydroélectriques),"** *Rev. gén. Hyd.*, 1947, vol. 13, May-June, pp. 134–142; July-Aug., pp. 171–190; Sept.-Oct., pp. 246–256; Nov.-Dec., pp. 301–316.

Little was known concerning pressure losses in very large water conduits when the Italian Committee for Big Hydroelectric Plants started their inquiries into the subject some years ago. It was found that the pressure losses in large pipes did not correspond with Nikuradse's laws (small pipes). A Swiss committee on big pipe lines wanted to study this, and the author investigated losses in 24 pipe lines on behalf of this committee. The tests were carried out with the utmost care and his report shows that pipes have to be divided into three categories:

1 Pipes with a character of roughness. These correspond to Nikuradse's laboratory-tested small pipes, and include entirely welded pipes with smooth internal transverse joints but with slightly rusted walls, all rusted welded pipes, and pipes with transverse riveted joints and very rough walls.

2 Pipes with character of smoothness. To this group belong new entirely welded rust-free pipes (which follow the law of Blasius) and pipes with riveted transverse joints, as long as the influence of the rivets is greater than that of the rusted walls (laws of Hopf).

3 This includes intermediary cases. This classification concerns only the character of the pipe, that is, the shape of the curve giving the friction factor λ (of Chézy) or n (of Manning) as a function of the Reynolds number R_e, but not the actual value of this factor. In all of these groups pipes can be rough or smooth according to the type of joint, the number of rows of rivets, and the condition of the walls. The actual values of λ (or n) are given for the 24 pipes tested.

Charles Jaeger, England

1285. **R. T. Knapp and A. Hollander, "Laboratory investigations of the mechanism of cavitation,"** *Trans. Amer. Soc. mech. Engrs.*, July 1948, vol. 70, pp. 419–435.

This publication is the first one to give the life history of a cavitation bubble, its growth, collapse, and rebound. The authors have developed a quantitative description of the actual physical processes which take place during cavitation. They make use of new experimental methods and equipment, consisting of a high-speed water tunnel and photographic recording of the details of the cavitation process by a new development in high-speed motion-picture photography, used to serve as a time microscope.

Their observations have been made on flow around bodies of revolution with varying ogive nose shapes, at velocities of from 30 to 70 fps with pressures approaching vapor pressure. Pictures have been taken at varying rates up to 20,000 per sec. The recording film moves constantly at a high speed, no camera shutter is present and the illumination is provided by synchronized flash lamps. Considerable development work has been carried on to increase the rate of 3000 flashes per sec (Edgerton) up to 50,000 by utilizing six control circuits discharging in rotation through a single lamp. The flash duration of one microsecond is of extreme intensity, corresponding to an energy input during the exposure of one flash at the rate of 1000 kw.

From the photographic records the authors have been able to carry on a detailed study of the formation, collapse, and rebound of an individual bubble, and to make calculations of rate of formation and collapse, from which deductions could be drawn concerning the physical mechanism of the cavitation phenomenon. They observed, after the first growth to maximum diameter and the first collapse to disappearance, three to five rebounds and recollapses of the same bubble before ultimate disappearance.

Strong inference is furnished that the pressure within the spherical bubble is approximately equal to the vapor pressure of the liquid at the mean temperature of the flow. Diagrams of bubble movement on time base, relation of bubble growth and collapse to pressure field, and volume radius of bubble on time base are given. The first growth to maximum diameter (0.28 in.) takes 0.0022 sec, from here to collapse 0.0008 sec. The authors make clear that the air pressure in the bubble is only negligible, although the water is saturated with air.

From the records a plot of the radial velocity of the bubble surface during the collapse period is given. This velocity V amounts to 765 fps at the time of impact. From thermal and kinetic considerations, and the very fact that rebound occurs to nearly the same size of bubble, there is strong evidence that the kinetic energy is stored in the liquid and given back essentially undiminished, this storage being accomplished by the common "water-hammer" phenomenon. The high localized pressure is calculated to be P (psi) $= 65V$ or approximately 50,000 psi.

The measured history of a bubble appears to be in striking agreement with Rayleigh's classical analysis of 1917. The authors show that there should be a scale effect in cavitation damage. The effect of clusters of bubbles is discussed. There is an interesting discussion and the authors' reply.

L. Troost, Holland

1286. **I. A. Charny, "On pressure variations upon sluice gates in a closed conduit with a surge tank in the tail water"** (in Russian), *Bull. Acad. Sci. USSR Ser. tech. Sci.* (*Izv. Akad. Nauk SSSR Ser. tekh. Nauk*), Aug. 1947, no. 8, pp. 929–944.

The author is concerned with the efficacy of a surge tank situated in the tail water just behind a sluice gate in a closed conduit, as a device for absorbing pressure pulsations in that region. The specific problem treated is that in which the pressure at a fixed distance downstream from the surge tank is assumed known as a function of time, and the pressure at the position of the surge tank is computed as a function of time. By using a Fourier integral representation for the given pressure, the author is able to find an integral giving the desired pressure function.

The case of a sudden jump in the applied pressure downstream is studied in some detail and the author concludes that the surge tank markedly reduces the pressure pulsations at the sluice gate, a result which he claims to be borne out by model experiments. The cause of the pulsations is not investigated.

J. V. Wehausen, USA

Marine Propulsion

(See also Revs. 1180, 1264, 1281)

1287. **Jean Dieudonne, "Marine propellers—results obtained at sea with cavitating propellers,"** *Shipbuilder*, Apr. 1948, vol. 55, no. 472, pp. 284–286.

Five series of trials show that under noncavitating conditions the thrust coefficient and the torque coefficient depend only on a single variable (the effective slip) which defines the kinematic condition. When cavitation occurs, the two coefficients become functions of two variables; one of them defines the kinematic condition, while the other must be a pressure parameter. Two diagrams plot the ratios λ_T and λ_P of the values of thrust or power under and without cavitation (at the same revolutions and slip) against a quantity proportional to the product of the slip and the pressure parameter. A last diagram shows the mean curves of λ_T and λ_P together with their ratio λ_ρ which defines a coefficient of efficiency.

L. J. Tison, Belgium

1288. C. W. Prohaska, "Residuary stability," *Trans. Instn. nav. Archit. Lond.*, 1947, vol. 89, pp. 342–375.

The author shows that the usual transverse stability calculations for ships are burdened with great uncertainty. To improve on the usual presentation of stability data, he introduces the concept of the "residuary stability" lever which is the part of the ordinary stability lever not depending on the metacentric height, or the distance from the initial metacentrum to the line of buoyancy, projected on a transverse plane. From a study of 12 geometrical forms, the author comes to the conclusion that form influence is rather unimportant in comparison with the influence of the proportions. From a statistical examination of residuary stability curves for 47 actual ships and their comparison with those derived from simpler forms, the author arrives at a very concentrated form of presentation of the results of his investigations. The stability diagram in Fig. 26 will be very useful for judging stability characteristics for ships of nonextreme forms at preliminary stages of design, as well as for checking of ordinary stability curves. There is a discussion and the author's reply.

L. Troost, Holland

1289. Jean Lefol, "Ship resistance—hull and propeller interaction," *Shipbuilder*, Apr. 1948, vol. 55, no. 472, pp. 265–268.

The relation between wake fraction and thrust deduction is discussed with regard to propulsion efficiency. Experimental means of obtaining the wake are discussed and various empirical formulas are summarized. A proposed method for determining wake fraction and thrust deduction is presented in the form of graphs of these in terms of five geometric parameters.

J. M. Robertson, USA

1290. E. Petersohn, "On the power obtainable from wind in sailing (Om den effekt, som vid segling utvinnes ur vinden)," *Tekn. Tidskr.*, June 19, 1948, vol. 78, pp. 401–404.

Formulas for the power obtainable from the wind in sailing are derived. Numerical results, computed by means of these formulas, are presented in a number of tables and diagrams, and their significance is discussed. Some particular examples are dealt with, including combinations of wind and engine power.

F. J. Plantema, Holland

1291. T. H. Havelock, "Calculations illustrating the effect of boundary layer on wave resistance," *Shipbuilder*, Apr. 1948, vol. 55, no. 472, pp. 268–272.

Theory leads to a wave resistance versus speed curve which oscillates rapidly and excessively at low speeds, at variance with experiment. An obvious explanation is that viscosity renders the stern of the ship less effective in wave production at low speeds. This reduction can be allowed for by an empirical factor, which leaves much to be desired from a theoretical point of view.

In this paper, the author examines the possible effect of boundary layer on wave resistance. Small modifications of the lines of a simple form near the stern simulate boundary-layer displacement. The ideal thin plank is considered first; then a form with simple parabolic lines and vertical sides; finally a calculation is made for a form which is unsymmetrical fore and aft in order to show the difference between the bow leading and the stern leading.

The results of the calculations show that the displacement effect of the boundary layer itself is negligible. However, the modification of ship form by rapid thickening of the boundary layer near the stern due to flow separation has a very material effect on the humps and hollows in the resistance curve. For all the forms considered, even relatively minor changes in form near the stern smoothed out or obliterated the humps in the resistance curves at low speeds. The hypothesis seems, therefore, well substantiated.

Karl E. Schoenherr, USA

1292. A. Emerson and L. W. Berry, "Experiments in the Lithgow Propeller Tunnel," *Shipbuilder*, Apr. 1948, vol. 55, no. 472, pp. 297–302.

Results of experiments on five related eight-inch marine propellers in the Lithgow Propeller Tunnel of the National Physical Laboratory are given and compared with data taken in an open tank. Various rpm and slip values are used and values of J (advance constant) run from 0.4 to 0.9. Thrust and torque coefficients and screw efficiencies are plotted against J at atmospheric pressure. Thrust and torque coefficients at reduced pressures are also plotted against the cavitation number, and the corresponding appearances of cavitation are sketched.

The superiority of a tunnel over an open tank is pointed out: hydrostatic pressures can be scaled as well as propeller sizes. Also, higher rpm values are attainable in the tunnel.

A. O. Williams, Jr., USA

Lubrication; Wear

1293. V. S. Schedrov, "Wear of a contact surface by compaction" (in Russian), *J. tech. Phys. (Zh. tekh. Fiz.)*, Apr. 1948, vol. 18, pp. 525–528.

In solids having a structure such that a nonelastic decrease of particle distance under load can occur, the shape of the surface is affected correspondingly. This process takes place in compaction of soil under pressure, or in the metal bushings of sliding bearings, such as sleeve bearings. The process leads to an increase in mean density, or a decrease in volume with constant mass. The corresponding effect on the shape of the contact surface is designated as "wear by compaction." It is assumed that of the two bodies in contact only one is of a material subject to this type of deformation.

General expressions are derived for the rate, as well as the amount of this type of wear, based on the "law of plastic and hereditary deformation" of Matsheret [*J. tech. Phys. (Zh. tekh. Fiz.)*, 1947, vol. 17, p. 87]. As a special case, metals are dealt with whose rate of strain hardening is an exponential function of stress. It is shown that even though compaction continues for an infinite length of time, the major portion occurs within a finite, short interval. All expressions derived by the author refer to a simple prism uniformly compressed at two opposite faces, and therefore do not strictly represent the case of contact pressure.

George Winter, USA

1294. M. C. Shaw and C. D. Strang, Jr., "The hydrosphere—a new hydrodynamic bearing," *J. appl. Mech.*, June 1948, vol. 15, pp. 137–145.

The paper purports to demonstrate analytically that, even if thermal and inertia effects are neglected, it is possible to generate a hydrodynamic, load-carrying film in a bearing, in the absence of the usual wedge-shaped passage in the direction of motion of the moving part, by making the oil flow normal to this direction through an appropriate inlet. A novel analysis employing the concept of a "Mean Stream Tube" is given, but does not yield an analytical determination of the pressure distribution.

The importance of the paper arises from its description of experiments on the friction and load-carrying capacity of thrust

bearings of special spherical form. Typical $\mu N/P$ curves are obtained, and it is demonstrated that this parameter alone does not determine the performance for this type of bearing, different curves being obtained for different speeds and viscosities.

The bearing will not operate satisfactorily unless it is provided with oil grooves of which there may be an optimum number. Material may be removed from the vicinity of the center of the bearing so that the shaft may be continued through the bearing. The hydrosphere design appears to offer some advantages over conventional bearing designs. F. T. Barwell, England

1295. **W. E. Campbell and Rose Kozak, "Studies in boundary lubrication—III. The wear of carbon brushes in dry atmospheres,"** *Trans. Amer. Soc. mech. Engrs.*, July 1948, vol. 70, pp. 491–498.

The authors measure the dusting wear of carbon brushes in dry nitrogen at atmospheric pressure, considering the effects of water, oxygen, and carbon dioxide, moderate current density, material of the rubbing surface, and crystal orientation. Their principal conclusion that proper crystal orientation prevents wear in dry nitrogen is questioned by a discusser on the basis of his experiments, and subsequently modified in the authors' closure.

F. J. Maginniss, USA

Dynamics of Meteorology and Oceanography

(See also Revs. 1179, 1278)

1296. **H. Riehl, "Jet stream in upper troposphere and cyclone formation,"** *Trans. Amer. geophys. Un.*, Apr. 1948, vol. 29, pp. 175–186.

Cases of relatively straight westerly flow aloft were investigated to find criteria indicating circumstances conducive to cyclone formation along low tropospheric disturbances in such instances. Pronounced cyclogenesis occured mainly if the westerly current aloft was very strong and concentrated in a narrow latitudinal band constituting a "jet stream."

The structure of the jet stream is discussed, with particular emphasis on the tropo-pause region. The suggested relation is illustrated by the period of January 27 to 29, 1947, held to be a typical example of more than ten cases of cyclone formation during the winter of 1946 and 1947.

It is suggested that the jet stream in connection with a pattern of very long waves in the Westerlies provides a mechanism for the initiation of cyclonic development and an increase in wave numbers. The jet is effective only if superimposed on a disturbance of the lower atmosphere. The suggested mechanism holds only for one species of cyclone formation.

Hans F. Winterkorn, USA

1297. **H. L. Penman, "Natural evaporation from open water, bare soil, and grass,"** *Proc. roy. Soc. Lond. Ser. A*, Apr. 22, 1948, vol. 193, pp. 120–145.

Natural evaporation from saturated surfaces (open water, bare soil, and grass) was approached theoretically (a) on an aerodynamic basis, regarding evaporation as due to turbulent transport of vapor by eddy diffusion, and (b) on an energy basis, regarding evaporation as one of the ways of degrading incoming radiation. A distinct contribution is the combination of approaches (a) and (b) which eliminates the parameter most difficult to measure (surface temperature), and provides a means of estimating evaporation rates from standard meteorological data.

Evaporation rates from wet bare soil and from turf with an adequate supply of water are obtained as fractions of the rate from open water; the fraction for turf shows a seasonal change attributed to the annual cycle of length of daylight. Experiments performed at Rothamsted are described, analyzed, and applied to account satisfactorily for open-water evaporation data obtained in Europe and America, bare-soil data from India, and turf data from the British Isles. Approximate average ratios for evaporation under equal conditions from open water, bare soil, and turf, respectively are 1, 0.9, and 0.75, however, the turf data refer specifically to southern England.

Hans F. Winterkorn, USA

1298. **R. B. Montgomery, "Viscosity and thermal conductivity of air and diffusivity of water vapor in air,"** *J. Met.*, Dec. 1947, vol. 4, pp. 193–196.

A summary and critical discussion is given of the best up-to-date values of viscosity and thermal conductivity of dry air and the diffusivity of water vapor in air. These would be used in dynamic meteorology. A list is also given of the more common atmospheric constants with probable errors. A list of references is appended covering the properties of air.

W. C. Johnson, Jr., USA

1299. **A. Pagnon and L. Romani, "On the statistical distribution of wind velocities in a given place (Sur la répartition statistique des vitesses du vent en un lieu donné),"** *C. R. Acad. Sci. Paris*, June 28, 1948, vol. 226, pp. 2170–2172.

The distribution of wind velocities is usually represented by a (θ, v) curve where θ is the fraction of the time during which wind velocities exceed v. The curve $(d\theta/dv, v)$ does not obey one simple law of Gibrat [$\log(v - v_o)$ proportional to z defined by $\theta = \pi^{-1/2} \int_0^z e^{-z^2} dz$]. In fact, it yields two such laws, represented by two straight lines meeting at an acute angle in the plane z, $\log (v - v_o)$. The first line corresponds to weak, local winds, while the second one refers to strong ones, originating in cyclonic perturbations. L. J. Tison, Belgium

Ballistics; Detonics (Explosions)

(See also Rev. 1256)

1300. **G. Birkhoff, D. P. MacDougall, E. M. Pugh, and G. Taylor, "Explosives with lined cavities,"** *J. appl. Phys.*, June 1948, vol. 19, pp. 563–582.

The authors describe the mechanism of the collapse of the wedge and conical-shaped cavity, and the resulting formation of a high-speed jet. For the conical-shaped cavity, the detonation of the booster starts an explosive wave down the charge, and, when this wave reaches the apex of the metallic-lined cone cavity, it suddenly produces pressures so large that the cone lining may be treated as though it were a perfect fluid. The high pressures on the outside cause the walls to move inward perpendicular to their surfaces, and at high velocities. The moving material retains a conical shape with apex moving outward along the axis. Behind the moving apex a section of thoroughly collapsed cone which contains material only from the outer part is found.

The inner part of the cone forms the jet squeezed out from the inner apex. This jet moves at a speed of 2000 to 10,000 meters per sec, and is solely responsible for the penetrating power of shaped charges with lined cavities. Its diameter is probably considerably less than a quarter of an inch. It is known to have appreciable wobble which accounts for its apparent larger diame-

ter. The metal from the outer surface of the cone forms into a slug and travels outward at 500 to 1000 meters per sec.

A mathematical theory is developed which predicts the velocities and masses of the jet and slug for both the conical and wedge-shaped liners. It is assumed that after the walls have received the original impulse, pressures on all sides of the liner quickly equalize, and the walls continue to collapse inward with no appreciable change in velocity. Bernoulli's equation is used to calculate the velocity from the pressure. It is pointed out that during collapse the jet and the slug will have exactly the same length, in agreement with experiment.

Measurements of the velocities agree quite well with the theory. The jet velocity increases as the conical angle decreases, and it is shown that it cannot exceed twice the detonation velocity. However, as the conical angle approaches zero, the mass of the jet tends to zero. Considerable discussion is given to the penetration of metal by jets, and a theory of penetration is developed. A curve of average penetration of mild steel as a function of standoff for CIT standard charges is given, as well as numerous illustrations and diagrams showing the various effects of the jets and their formations. A bibliography is included.

C. O. Dohrenwend, USA

Thermodynamics

(See also Revs. 1248, 1249, 1253, 1274, 1297, 1304)

1301. Walter T. Olson and Everett Bernardo, "Temperature measurements and combustion efficiency in combustors for gas-turbine engines," *Trans. Amer. Soc. mech. Engrs.*, May 1948, vol. 70, pp. 329–334.

This paper describes in detail a thermocouple installation used successfully to examine the behavior of several gas-turbine combustors. It is shown that a few common bare-wire thermocouples adequately determine the state of the air entering the combustor, while it appears that nonuniformities and fluctuations of velocities and temperatures at the combustor outlet will introduce appreciable errors into the measurements unless a large number of probes are used. Some altitude performance curves for an annular-type combustor are also presented.

Andrew Fejer, USA

1302. G. A. Hawkins, W. L. Sibbitt, and H. L. Solberg, "Review of data on dynamic viscosity of water and superheated steam," *Trans. Amer. Soc. mech. Engrs.*, Jan. 1948, vol. 70, pp. 19–23.

The authors present a compilation of data on the viscosities of water and steam, as obtained by Sigwart, Schiller, Speyerer, Schougayew, Timroth, and Hawkins and coworkers, and of the experimental methods employed. Some reasons for the great discrepancies in these data are suggested. The need for further research is emphasized.

E. F. Lype, USA

Heat Transfer

(See also Revs. 1238, 1276, 1298)

1303. H. Parodi, "Heating by solar radiation of containers closed by absorbing walls (Note sur l'échauffement produit par le rayonnement solaire dans des enceintes closes par des parois absorbantes)," *C. R. Acad. Sci. Paris*, June 28, 1948, vol. 226, pp. 2128–2129.

The author shows, from empirical data on fuel consumption and on monthly average temperatures and solar radiation, that the 25 to 30 per cent discrepancy between "theoretical" (based simply on the "degree-day" concept) and actual heating requirements can be accounted for by the absorption of solar radiation.

Stanley Corrsin, USA

1304. J. R. Stalder and D. Jukoff, "Heat transfer to bodies traveling at high speed in the upper atmosphere," *J. aero. Sci.*, July 1948, vol. 15, pp. 381–391.

The authors calculate equilibrium temperatures for flat plates moving at speeds from 0 to 20,000 fps, attack angles from 0 to 20 deg, altitudes from 75 to 150 miles, with or without solar radiation and internal heat transfer. For the altitudes considered, the molecular mean free path in the atmosphere is so large that customary aerodynamics is invalid, and the authors' results can be generalized to arbitrarily shaped bodies by simple integration.

The principal conclusions follow:

1 With solar radiation absent, the skin-cooling problem is negligible for altitudes $\geq$ 100 miles, speeds $\leq$ 20,000 fps, and attack angles $\geq$ 20 deg.

2 The effect of solar radiation on skin temperature is small at 75 miles altitude—predominant at 150 miles.

3 High fineness ratio and much surface at negative attack angles lessen heating effect.

H. G. Elrod, Jr., USA